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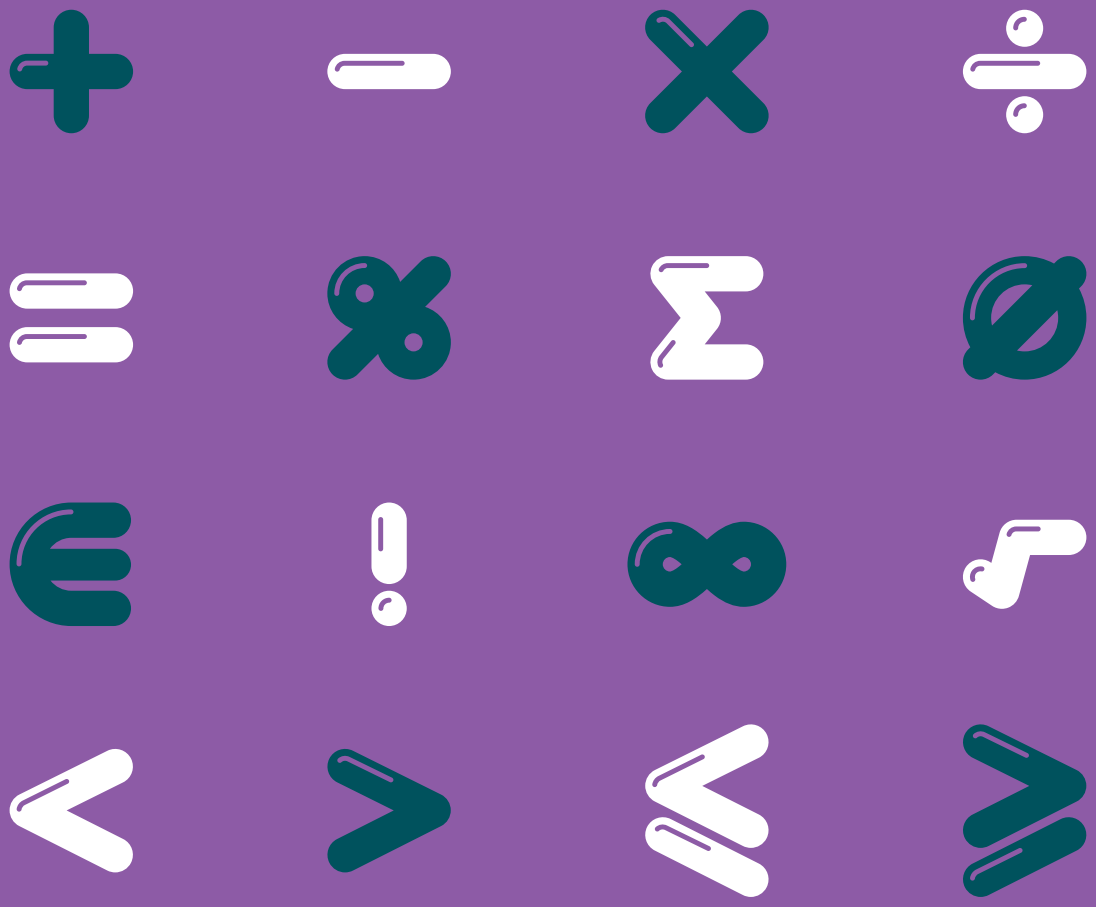

# MATHEMATICS STAGE 5

## DIAGNOSTIC TASKS

# MATHEMATICS STAGE 5

# Introduction

Mathematics Stage 5 Diagnostic Tasks have been designed to assist secondary teachers in accurately identifying skills, knowledge and understandings of students in Stage 5. They are intended to be a diagnostic snapshot of learning key ideas and concepts from the NSW Mathematics K-10 Syllabus for the Australian Curriculum.
This resource is one example of assessment for learning; other forms of assessment include hands-on tasks, practical activities, investigations, observations and anecdotal evidence. A wide range of assessments are also used by teachers to identify teaching points and learning needs for all students.

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Mathematics Stage 5 Diagnostic Tasks


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# MATHEMATICS STAGE 5.1

# Mathematics Stage 5.1: Financial Mathematics

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: COMPUTATION WITH INTEGERS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Order integers | |
| ☐ Add and subtract integers | |
| ☐ Multiply and divide integers | |
| ☐ Perform order of operations | |
| ☐ Use a calculator to perform operations with integers | |
| ☐ Solve problems | |

### NUMBER AND ALGEBRA: FRACTIONS, DECIMALS AND PERCENTAGES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Express improper fractions as mixed numerals and vice versa | |
| ☐ Generate equivalent fractions, write fractions in simplest form | |
| ☐ Apply the four operations with fractions and decimals | |
| ☐ Convert between fractions, decimals and percentages | |
| ☐ Express one quantity as a fraction/percentage of another | |
| ☐ Round decimals to a specified number of places | |
| ☐ Use the notation of recurring decimals | |
| ☐ Convert fractions into recurring decimals | |
| ☐ Solve problems involving fractions, decimals and percentages | |

### NUMBER AND ALGEBRA: FINANCIAL MATHEMATICS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Perform calculations involving GST | |
| ☐ Calculate discounts and 'best buys' | |
| ☐ Solve problems involving profit and loss | |

Name: _____

Class: _____

## STAGE 5.1: FINANCIAL MATHEMATICS

### SYLLABUS REFERENCE

**MA5.1- 4NA** - A student solves financial problems involving earning, spending and investing money.

### QUESTION 1 — SOLVE PROBLEMS INVOLVING EARNING MONEY

a) Lucy works a 40 hour week and gets paid $13.50 an hour. What is her total earnings for the week?

_____

b) Julie works 28 hours as a beautician. She earns $365.50 a week. What is her hourly rate of pay?

_____

c) Chris earns $16.30 per hour for a 36 hour week. He receives overtime at time and a half for the first 4 hours and double time thereafter. Calculate Chris's earnings for the week if he works 45 hours.

_____

_____

_____

_____

d) Craig is a builder and earns a salary of $76 500 p.a. Find Craig's:

i) fortnightly pay _____

ii) monthly pay _____

e) Amirita is a computer programmer and earns a salary of $67 500. She decides to take her annual holidays and is paid 4 weeks normal pay plus 17.5% holiday loading.
*(Holiday loading is calculated on 4 weeks normal pay)*

i) Calculate her holiday loading.

_____

ii) Calculate her total holiday pay.

_____

## QUESTION 2

**COMMISSION AND PIECE WORK**

a) John is a car salesman. He earns a retainer of $350 a week plus 3% commission on the value of car sales exceeding $25 000. John sold a total of $55 000 in car sales this week. What is his total pay for the week?

b) A factory worker earns $1.30 for every wooden toy he makes and 45c for every plastic toy he makes. If he makes 40 wooden toys and 20 plastic toys. How much would he receive altogether?

## QUESTION 3

**SOLVE PROBLEMS INVOLVING TAXATION**

a) Andrew earns an annual gross salary of $47 560. His employer deducts $15 000 in tax, $550 in union fees and $6 543.67 for superannuation per year. What is his annual net income?

b) The tax table below shows the tax rates for 2016.

| TAXABLE INCOME | TAX PAYABLE |
|---|---|
| 0 - $18 200 | Nil |
| $18 201 - $37 000 | 19c for each $1 over $18 200 |
| $37 001 - $80 000 | $3 572 plus 32.5c for each $1 over $37 000 |
| $80 001 - $180 000 | $17 547 plus 37c for each $1 over $80 000 |
| $180 001 and over | $54 547 plus 45c for each $1 over $180 000 |

i) Audrey earns a salary of $46 540. How much tax does she pay?

ii) Audrey's employer has already taken out $7750 in PAYG tax instalments this year. Is she entitled to a refund or is she liable to pay tax back? What is her refund or liability amount?

iii) Audrey is required to pay a Medicare levy of 1.5% on her taxable income.
Calculate the Medicare Levy she is required to pay.

**QUESTION 4**

**APPLY THE SIMPLE INTEREST FORMULA TO SOLVE PROBLEMS INCLUDING BUYING ON TERMS**

a) Find the simple interest earned on an investment of $6500 at 6% p.a. for 4 years.

b) Rob borrowed $3540 for 3 years and was charged simple interest at 2.5% p.a. How much did he repay altogether including interest?

c) Mary took out a student loan of $1500 for 7 months at 7.5% p.a. Calculate the simple interest on her loan.

d) Fadila purchased a television for her room for $5400. She paid an initial deposit of $1500 and borrowed the rest at 12.8% p.a. simple interest for 2 years.

Calculate:

i) the amount borrowed.

ii) the simple interest charged on the amount borrowed.

iii) the total to be repaid.

iv) the total price paid for the television.

**QUESTION 5**

**CALCULATE COMPOUND INTEREST USING REPEATED APPLICATIONS OF SIMPLE INTEREST**

a) Lisa invested $3000 at 7% p.a. compounded over 3 years.

Calculate:

i) the total of her investment at the end of 3 years.

ii) the compound interest earned.

b) Use the repeated multiplication method to find the final value of $5400 invested at 6.3% p.a. for 3 years.

c)

| COMPOUND INTEREST TABLE<br>FINAL AMOUNT (A) OF $1 INVESTMENT | | | | |
|---|---|---|---|---|
| NUMBER OF PERIODS (N) | INTEREST RATE PER COMPOUNDING PERIOD (R) | | | |
| | 2%<br>0.02 | 2.5%<br>0.025 | 3%<br>0.03 | 3.5%<br>0.035 |
| 1 | 1.02 | 1.025 | 1.03 | 1.035 |
| 2 | 1.0404 | 1.0506 | 1.0609 | 1.0712 |
| 4 | 1.0824 | 1.1038 | 1.1255 | 1.1475 |
| 6 | 1.1262 | 1.1597 | 1.1941 | 1.1877 |

Use the compound interest table above to calculate the total investment for the following:

i) $5400 invested at 3% p.a. compounded yearly for 2 years.

ii) $4500 invested at 6% p.a. compounded half yearly for 3 years.

d) Compare the following investments. Which is the better investment? (show working)

**INVESTMENT A**

$5000 invested at a simple interest rate at 3% p.a.
for 6 years

**INVESTMENT B**

$5000 invested at 3% p.a.
Compounded yearly for 6 years
(Use the compound interest table)

# Mathematics Stage 5.1: Indices

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: INDICES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Use index notation for positive integral indices | |
| ☐ Express a whole number as a product of its prime factors | |
| ☐ Apply the order of operations to evaluate indices | |
| ☐ Determine and apply tests of divisibility for 2, 3, 4, 5, 6, 10 | |
| ☐ Determine and apply the index laws for numerical expressions and the meaning of the zero index | |
| ☐ Verify the index laws using a calculator | |

## STAGE 5.1: INDICES

### SYLLABUS REFERENCE

**MA5.1-5NA** - A student operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases.

### QUESTION 1

**APPLY THE INDEX LAWS TO SIMPLIFY ALGEBRAIC EXPRESSIONS WHEN MULTIPLYING INDICES**

a) Write the following in index notation:

i) $a \times a \times a \times a =$ _____

ii) $2 \times 2 \times 2 \times 2 \times 2 =$ _____

b) Simplify the following:

i) $2^3 \times 2^2 =$ _____

ii) $m^2 \times m^3 \times m^4 =$ _____

iii) $2x^2 \times 3x^4 =$ _____

iv) $-p^2 \times (-5p^2) =$ _____

v) $4xy^2 \times 8x^2y^3 =$ _____

**QUESTION 2** **APPLY INDEX LAWS WHEN DIVIDING INDICES**

Simplify the following:

a) $y^5 \div y^2 =$ _____

b) $8x^2 \div 4x =$ _____

c) $30k^3 \div -5k^2 =$ _____

d) $\dfrac{24x^3y^5}{6x^2y^3} =$ _____

**QUESTION 3** **APPLY INDEX LAWS WHEN DIVIDING INDICES**

Simplify the following, giving answers in index form:

a) $(5^3)^2 =$ _____

b) $(2q^3)^3 =$ _____

c) $\left(\dfrac{7c}{d}\right)^2 =$ _____

**QUESTION 4** **THE ZERO INDEX**

Simplify the following:

a) $p^0 =$ _____

b) $2x^0 =$ _____

c) $(-5p)^0 =$ _____

## QUESTION 5

**DEFINE AND USE NEGATIVE-INTEGER INDICES TO EVALUATE NUMERICAL EXPRESSIONS**

a) Rewrite the following with a positive index:

i) $2^{-2} =$ _____

ii) $5^{-3} =$ _____

b) Write the following with negative indices:

i) $\frac{1}{2^4} =$ _____

ii) $-\frac{1}{4^3} =$ _____

c) Evaluate the following: (giving your answers as fractions)

i) $3^{-2} =$ _____

ii) $-5^{-3} =$ _____

iii) $-\frac{1}{4^2} + 2^{-2} =$ _____

# Mathematics Stage 5.1: Linear Relationships

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: LINEAR RELATIONSHIPS AND NON-LINEAR RELATIONSHIPS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Locate and describe points on the Cartesian plane using coordinates | |
| ☐ Describe translations and reflections in an axis on the Cartesian plane | |
| ☐ Describe rotations of multiples of 90° on the Cartesian plane | |
| ☐ Recognise, describe and record geometric and number patterns in words and algebraic symbols | |
| ☐ Plot linear relationships created from simple patterns and equations | |
| ☐ Solve simple linear equations using graphical techniques | |

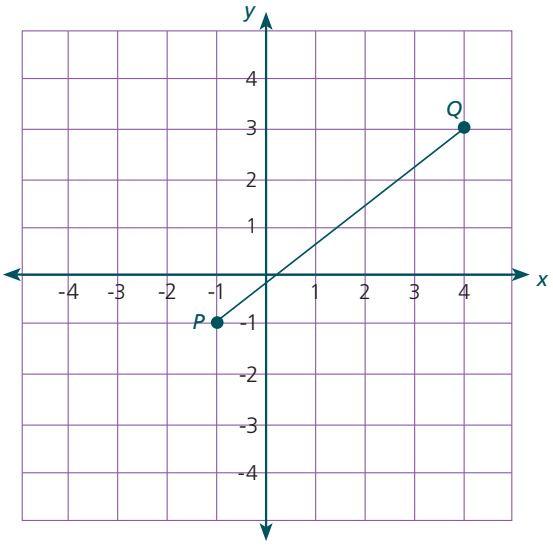
## STAGE 5.1: LINEAR RELATIONSHIPS

### SYLLABUS REFERENCE

**MA5.1-6NA** - A student determines the midpoint, gradient and length of an interval, and graphs linear relationships.

## QUESTION 1

**FIND THE MIDPOINT, GRADIENT AND LENGTH OF INTERVALS ON THE CARTESIAN PLANE USING INFORMAL STRATEGIES**




a) Does the interval PQ have a positive or negative gradient? _____

b) Determine the midpoint, gradient and length for the interval joining P (-1,-1) and Q (4,3) on the Cartesian plane.

i) Midpoint _____

ii) Gradient _____

iii) Length _____

## QUESTION 2

**GRAPH LINEAR RELATIONSHIPS FROM EQUATIONS**

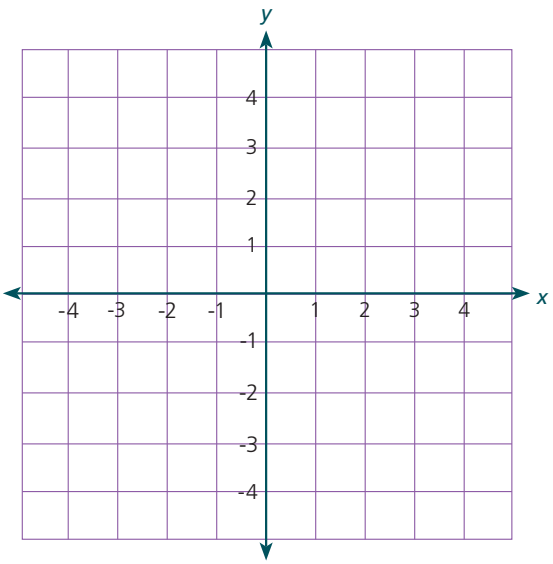
Graph the following linear relationships on the Cartesian plane.

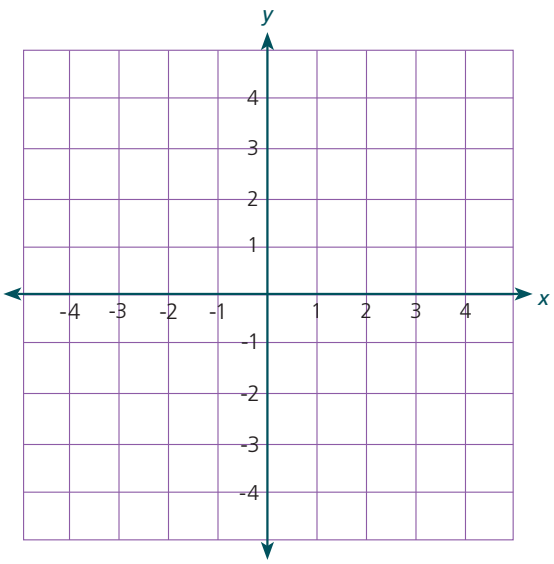
a) $x = 2$

| $x$ | | | | | | |
|---|---|---|---|---|---|---|
| $y$ | -2 | -1 | 0 | 1 | 2 | 3 |

b) $y = -2$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |



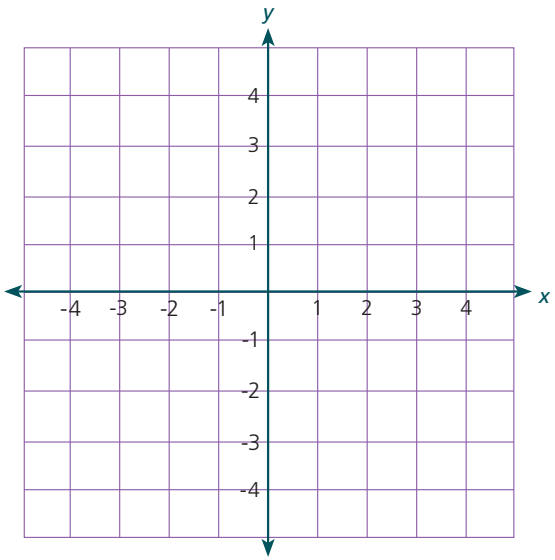
## QUESTION 3

GRAPH LINEAR RELATIONSHIPS FROM EQUATIONS

For the linear equation $y = x + 3$:

a) identify the $x$ and $y$ intercepts.

b) graph the linear equation $y = x + 3$.




## QUESTION 4

DETERMINE WHETHER A POINT LIES ON A LINE BY SUBSTITUTION

Does the point (2,3) lie on the line $y = 3x - 2$. (show working)

## QUESTION 5

DETERMINE THAT PARALLEL LINES HAVE EQUAL GRADIENTS

From the list below circle the lines that are parallel.

$y = 3x$ $y = 2x + 3$ $y = 3x + 1$ $y = -3x$

# Stage 5.1: Non-Linear Relationships

Name: _____

Class: _____

## STAGE 5.1: NON-LINEAR RELATIONSHIPS

### SYLLABUS REFERENCE

**MA5.1-7NA** - A student graphs simple non-linear relationships.

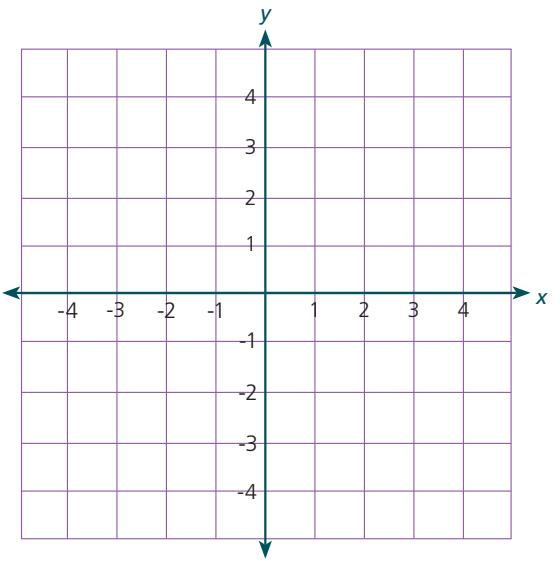
### QUESTION 1

**GRAPH SIMPLE PARABOLAS AND EXPONENTIALS ON THE CARTESIAN PLANE USING TABLES OF VALUES**

Graph the following non-linear relationships on the Cartesian plane.

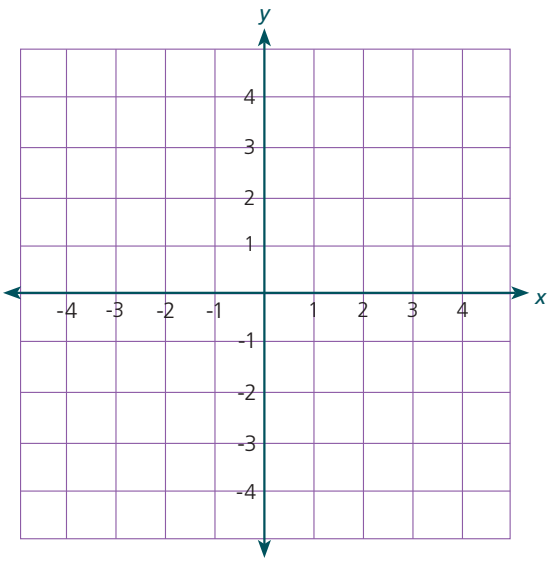
i) $y = x^2$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |




ii) $y = 2^x$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |



## QUESTION 2

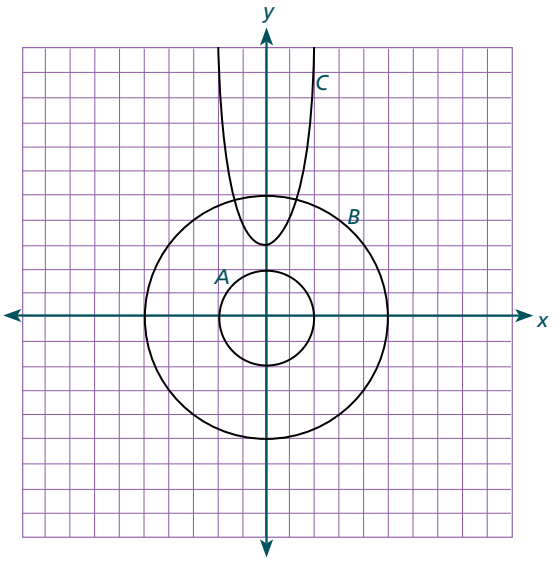
**CONNECT THE SHAPE OF A NON-LINEAR GRAPH WITH THE DISTINGUISHING FEATURES OF ITS EQUATION**

Match the following equations to the graphs A, B and C.

i) $x^2 + y^2 = 4$ _____

ii $y = x^2 + 3$ _____

iii) $x^2 + y^2 = 25$ _____




## QUESTION 3

**COMPARE NON-LINEAR RELATIONSHIPS**

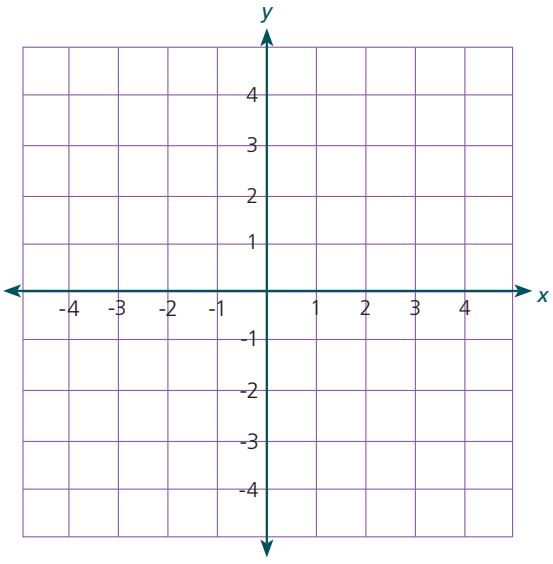
i) Sketch the graph y = $x^2$ + 1 and $y = x^2 - 1$ on the same number plane. Comment on where the graphs cross the y axis.

$y = x^2 + 1$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

$y = x^2 - 1$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |




ii) What effect does the constant have on each of the graphs?

_____

_____

_____

# Mathematics Stage 5.1: Area and Surface Area

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### MEASUREMENT AND GEOMETRY: AREA AND SURFACE AREA

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Convert between metric units of area | |
| ☐ Establish and use formulas to find the areas of triangles, special quadrilaterals and circles | |
| ☐ Solve problems involving area | |

## STAGE 5.1: AREA AND SURFACE AREA

### SYLLABUS REFERENCE

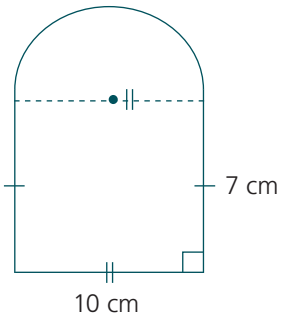
**MA5.1-8MG** - A student calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms.

### QUESTION 1

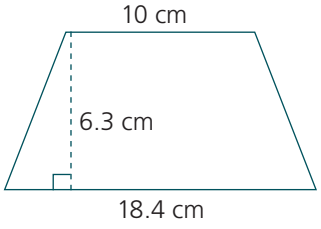
**SOLVE PROBLEMS INVOLVING AREAS OF COMPOSITE SHAPES BY DISSECTION INTO TRIANGLES, QUADRILATERALS, QUADRANTS, SEMI CIRCLES AND SECTORS**

Calculate the area of the following:

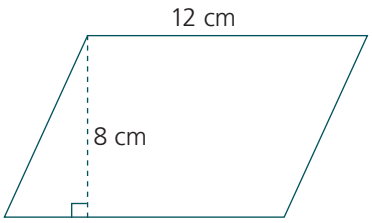
a)



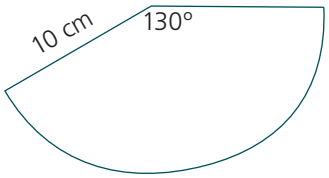

b)



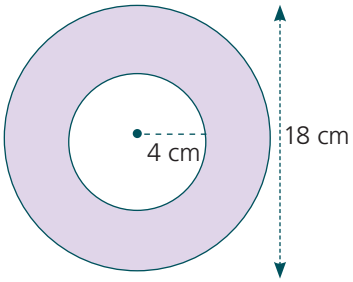
c)



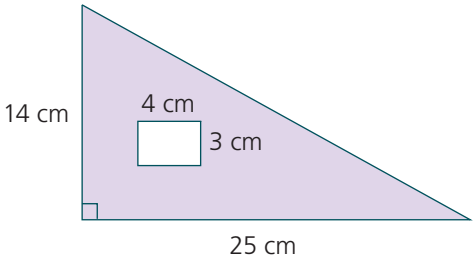

d)




e) calculate the shaded area.



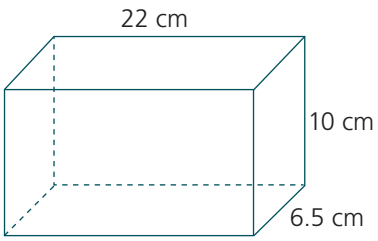

f) calculate the shaded area.



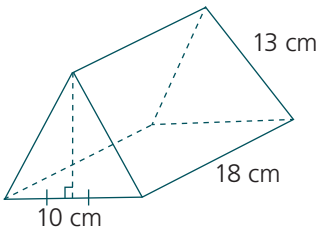
## QUESTION 2 SOLVE PROBLEMS INVOLVING THE SURFACE AREA OF RECTANGULAR AND TRIANGULAR PRISMS

Calculate the surface area of the following:

a)




b)



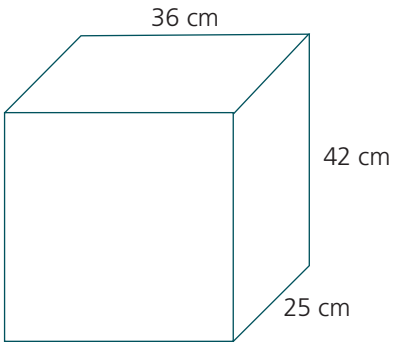

(*Hint you will need to use Pythagoras' Theorem*)

## QUESTION 3 SOLVE PROBLEMS INVOLVING THE SURFACE AREA OF RECTANGULAR AND TRIANGULAR PRISM

The diagram below shows a box which is open at the top:

a) How many faces does the box have?

b) Calculate the surface area of the box.



# Mathematics Stage 5.1: Numbers of Any Magnitude

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### MEASUREMENT AND GEOMETRY: NUMBERS OF ANY MAGNITUDE

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Perform operations with time units using mental computations and with a calculator | |
| ☐ Interpret international time zones | |

## STAGE 5.1: NUMBERS OF ANY MAGNITUDE

### SYLLABUS REFERENCE

**MA5.1-9MG** - A student interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures.

### QUESTION 1 — ROUND NUMBERS TO A SPECIFIED NUMBER OF SIGNIFICANT FIGURES

Round the following to 3 significant figures:

a) 2432 _____

b) 0.237541 _____

c) 1.287852 _____

## QUESTION 2

**CONVERT BETWEEN METRIC UNITS FOR VERY LARGE AND VERY SMALL MEASUREMENTS, INCLUDING MEASUREMENTS OF DIGITAL INFORMATION**

a) Convert the following:

i) 0.0078 m to cm _____

ii) 0.00234 mm to m _____

iii) 1860 nm to m _____

iv) 0.006 cm to $\mu m$ _____

b) Convert the following:

i) 7850 000 mL to L _____

ii) 0.00007826 kL to mL _____

c) Convert the following:

i) 7 MB to KB _____

ii) 5200 GB to TB _____

iii) 2.1 TB to KB _____

d) Convert the following:

i) 7.8 minutes to milliseconds _____

ii) 170000 microseconds to minutes _____

iii) 1 fortnight to seconds _____

iv) 897 years to decades _____

## QUESTION 3

**EXPRESS NUMBERS IN SCIENTIFIC NOTATION USING POSITIVE AND NEGATIVE POWERS OF 10**

a) Express the following in scientific notation:

i) 2800 _____

ii) 1 240 000 _____

iii) 0.03456 _____

iv) 0.0000541 _____

b) Express each number in decimal form:

i) $2.6 \times 10^{2}$ _____

ii) $4.56 \times 10^{-4}$ _____

**QUESTION 4** **ORDER NUMBERS EXPRESSED IN SCIENTIFIC NOTATION**

Write each set of numbers in ascending order.

a) $1.2 \times 10^{9}$, $6.3 \times 10^{8}$, $3.5 \times 10^{9}$ _____

b) $4.1 \times 10^{-2}$, $3.8 \times 10^{-1}$, $4.8 \times 10^{-3}$ _____

**QUESTION 5** **SOLVE PROBLEMS INVOLVING SCIENTIFIC NOTATION**

a) The Earth is $1.52 \times 10^{8}$ kilometres from the Sun. If the speed of light is $3 \times 10^{5}$ kilometres per second, how long does it take light to travel from the Sun to Earth? Give your answer to the nearest second.

_____

_____

b) Rachael is an author. She types 28 000 words every day. How many words will she type in a year? (answer in scientific notation)

_____

**QUESTION 6** **LIMITS OF ACCURACY**

Lisa weighs her dog and records its weight as 16kg. What:

a) is the limit of accuracy of Lisa's measurement?

_____

b) could be the actual weight of Lisa's dog?

_____

**QUESTION 7** **DEGREE OF ACCURACY**

Round the following to the appropriate degree of accuracy.

a) 26.1 + 13.45 = _____

b) 140 + 18.1 = _____

# Mathematics Stage 5.1: Right-Angled Triangles (Trigonometry)

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### MEASUREMENT AND GEOMETRY: TRIGONOMETRY

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Establish and apply Pythagoras' theorem to find sides in right-angled triangles | |
| ☐ Solve problems involving Pythagoras' theorem | |

## STAGE 5.1: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

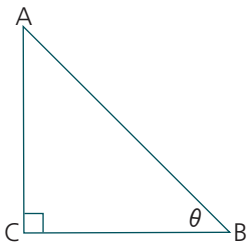
### SYLLABUS REFERENCE

**MA5.1-10MG** - A student applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression

### QUESTION 1

**USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (ANGLES MEASURED TO NEAREST DEGREE)**

a) In $\Delta ABC$ identify the following sides with respect to $\theta$:

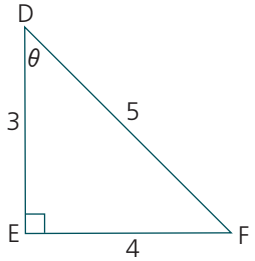



i) hypotenuse _____

ii) adjacent side _____

iii) opposite side _____

b) Find the value for sin $\theta$, cos $\theta$ and tan $\theta$ in $\Delta DEF$:




i) sin $\theta$ _____

ii) cos $\theta$ _____

iii) tan $\theta$ _____

c) Evaluate the following to 2 decimal places:

i) cos 73° = _____

ii) tan 12° = _____

iii) 3 x sin23° = _____

d) Find $\theta$ to the nearest degree:

i) cos $\theta$ = 0.143 _____

ii) tan $\theta$ = 1.56 _____

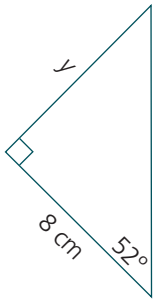
iii) sin $\theta = \frac{3}{5}$ _____

**QUESTION 2**

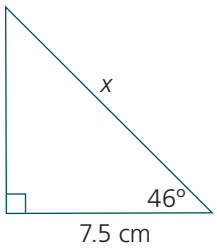
**USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (ANGLES MEASURED TO NEAREST DEGREE)**

a) Find the value of the pronumeral in the following:

i)




ii)



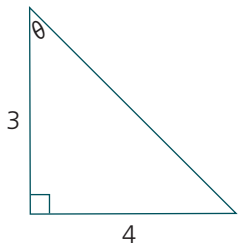

_____ _____

_____ _____

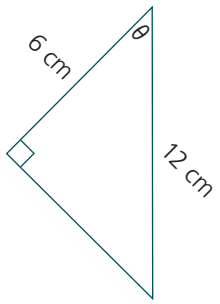
_____ _____

b) Find the value of $\theta$ in the following to the nearest degree:

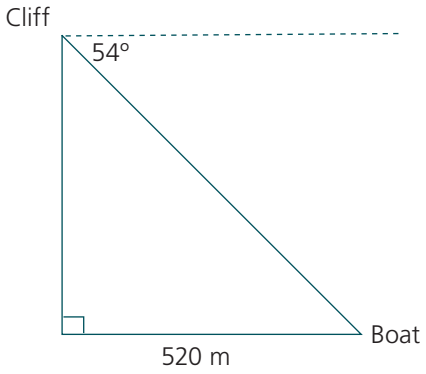
i)




ii)




c) The angle of depression of a boat from the top of the cliff is 54°. If the boat is 520 m from the base of the cliff, calculate the height of the cliff? (answer to the nearest metre)



# Mathematics Stage 5.1: Properties of Geometric Figures

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### MEASUREMENT AND GEOMETRY: PROPERTIES OF GEOMETRIC FIGURES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Classify and determine properties of triangles and quadrilaterals | |
| ☐ Identify line and rotational symmetries | |
| ☐ Determine the angle sums of triangles and quadrilaterals | |
| ☐ Use properties of shapes to find unknown sides and angles in triangles and quadrilaterals giving a reason | |
| ☐ Identify congruent figures | |
| ☐ Identify congruent triangles using the four tests | |

## STAGE 5.1: PROPERTIES OF GEOMETRIC FIGURES

### SYLLABUS REFERENCE

**MA5.1-11MG** - A student describes and applies the properties of similar figures and scale drawings.

### QUESTION 1 — DESCRIBE SIMILAR FIGURES

Explain what makes two shapes similar. _____

_____

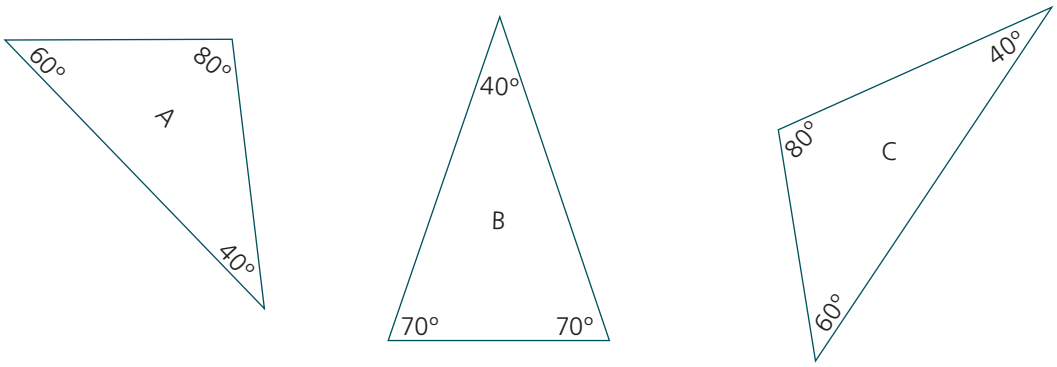
_____

## QUESTION 2

**IDENTIFY SIMILAR FIGURES AND DESCRIBE THEIR PROPERTIES INFORMALLY**

Which two pairs of triangles below are similar? _____

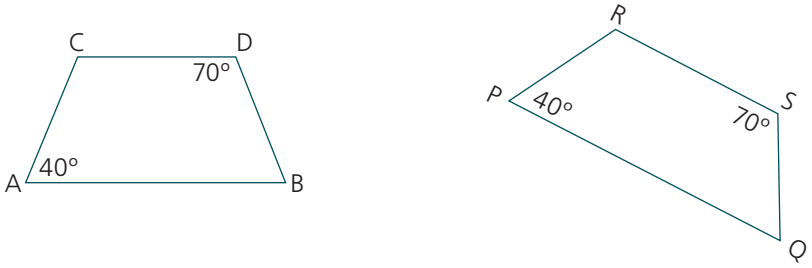
How do you know? _____




## QUESTION 3

**IDENTIFY SIMILAR FIGURES AND DESCRIBE THEIR PROPERTIES INFORMALLY**

Using the two similar trapeziums below:




a) name the four pairs of matching sides.

_____

b) name the four pairs of matching angles.

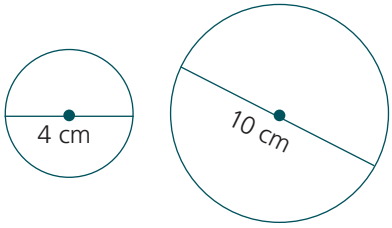
_____

## QUESTION 4

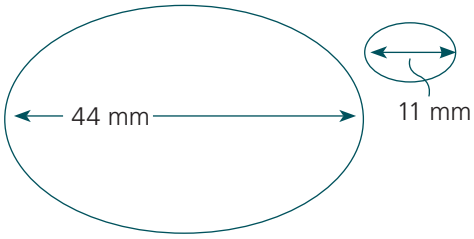
**USE SCALE FACTORS AND PROPORTION STATEMENTS TO DETERMINE UNKNOWN LENGTHS IN SIMILAR FIGURES**

a) Find the scale factor for the similar figures below:

i)



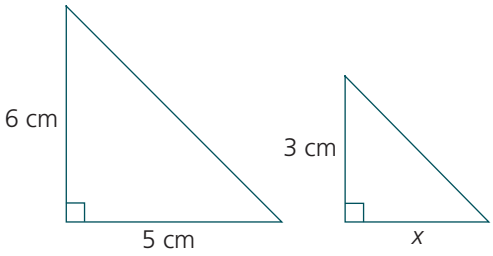

ii)




b) For the similar figures below find:

i) the scale factor

ii) the length of the unknown side




## QUESTION 5

**USE SCALES ON MAPS, PLANS AND DRAWINGS**

James has a small photograph of a door that he wants to enlarge to have a length of 34 cm.



i) What is the scale used?

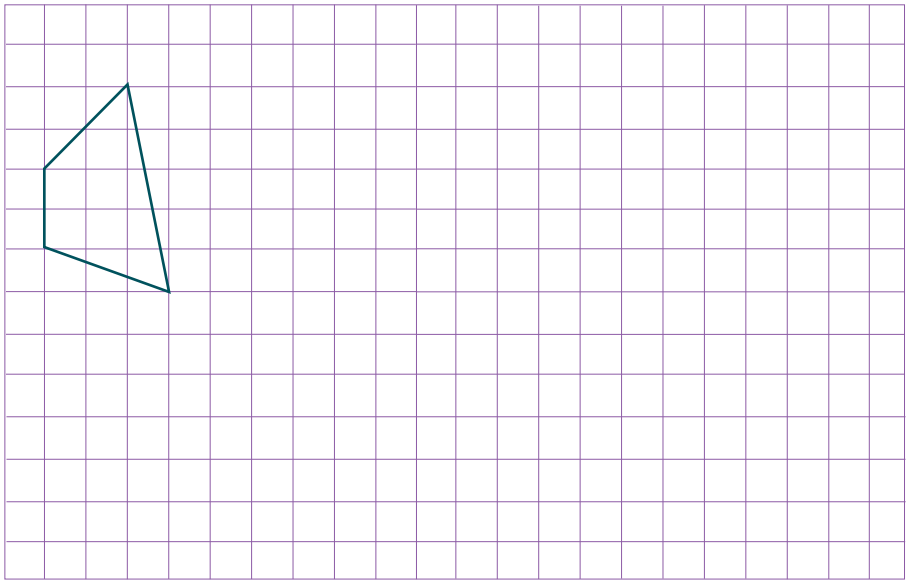
ii) Hence, find the real width of the door.

## QUESTION 6

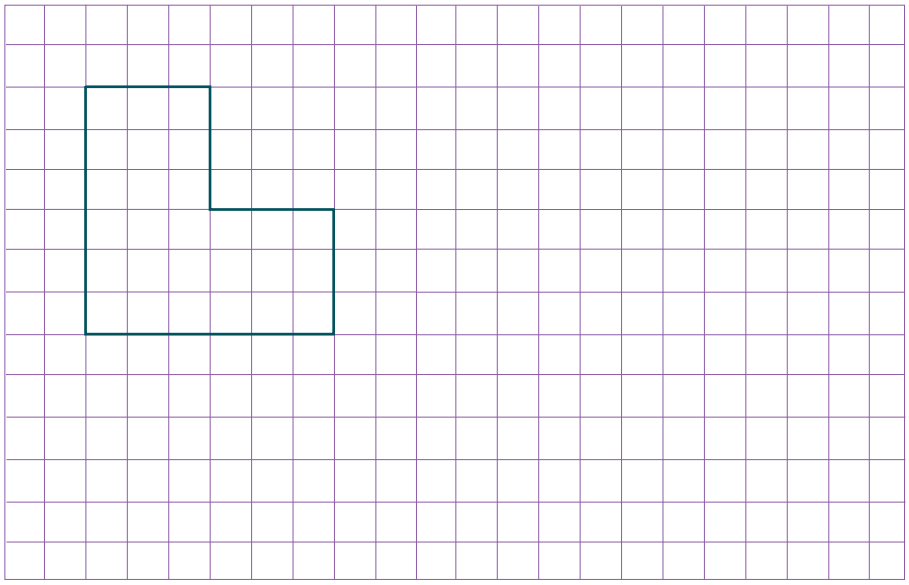
CONSTRUCT SCALE DRAWINGS

Draw the image of the figures below using the given scale factor.

a) Scale factor = 2



b) Scale factor = $\frac{1}{3}$



# Mathematics Stage 5.1: Single Variable Data Analysis

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### STATISTICS AND PROBABILITY: SINGLE VARIABLE DATA ANALYSIS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Calculate mean, median, mode and range for sets of data | |
| ☐ Investigate the effect of outliers on the mean and median | |
| ☐ Describe and interpret a variety of data displays using mean, median and range | |
| ☐ Calculate and compare summary statistics of different samples drawn from the same population | |

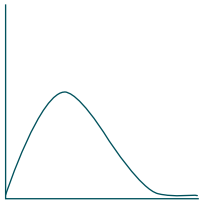
## STAGE 5.1: SINGLE VARIABLE DATA ANALYSIS

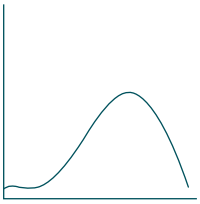
### SYLLABUS REFERENCE

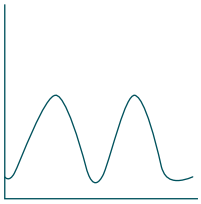
**MA5.1-12SP** - A student uses statistical displays to compare sets of data, and evaluates statistical claims made in the media.

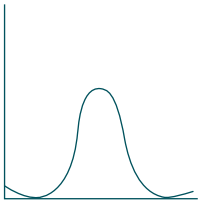
### QUESTION 1 — DESCRIBE DATA, USING TERMS INCLUDING 'SKEWED', 'SYMMETRIC' AND 'BI-MODAL'

Describe the following data representations as positively skewed, negatively skewed, symmetric or bi-modal.









_____ _____ _____ _____

_____ _____ _____ _____

## QUESTION 2

**CONSTRUCT AND INTERPRET BACK-TO-BACK STEM AND LEAF PLOTS**

The results of a mathematics exam are given below for a class of year 9 students:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **GIRLS:** | 9 | 22 | 24 | 25 | 33 | 36 | 37 | 40 |
| **BOYS:** | 5 | 6 | 12 | 25 | 26 | 33 | 34 | 35 |

i) Complete the following back to back stem and leaf plot to display the above results.

**MATHEMATICS RESULTS**

| | LEAF |
|---|---|
| | 0 |
| | 1 |
| | 2 |
| | 3 |
| | 4 |

ii) Determine whether a boy or a girl achieved the lowest score?

_____

iii) Use the mean, median and range to compare the two data sets.
Who achieved the better results overall, boys or girls?

_____

_____

iv) Comment on the skewness of the data for the boys' results.

_____

## QUESTION 3

**INTERPRET AND CRITICALLY EVALUATE REPORTS IN THE MEDIA AND ELSEWHERE THAT LINK CLAIMS TO DATA DISPLAYS AND STATISTICS**

Interpret and comment on the following statistical statements made in the media.

i) Data collected about Facebook and Twitter states that 25% of users on social media do not bother with privacy settings.

_____

_____

ii) According to the 2006 Census, about 36% of Ultimo residents walked to work.

_____

_____

# Mathematics Stage 5.1: Probability

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### STATISTICS AND PROBABILITY: PROBABILITY

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Construct sample spaces for single-step experiments with equally likely outcomes | |
| ☐ Find probabilities of events in single-step experiments | |
| ☐ Identify complementary events and use the sum of probabilities to solve problems | |
| ☐ Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and' | |
| ☐ Represent events in two-way tables and Venn diagrams and solve related problems | |

## STAGE 5.1: PROBABILITY

### SYLLABUS REFERENCE

**MA5.1-12SP** - A student uses statistical displays to compare sets of data, and evaluates statistical claims made in the media.

### QUESTION 1 — CALCULATE RELATIVE FREQUENCIES TO ESTIMATE PROBABILITIES

The following frequency table shows the results of data collected from 50 people on how they travel to work.

| TRANSPORT | FREQUENCY |
|---|---|
| Car | 4 |
| Train | 20 |
| Bus | 16 |
| Ferry | 5 |
| Walk | 2 |
| Bicycle | 3 |

i) Calculate the relative frequency for travelling by car. _____

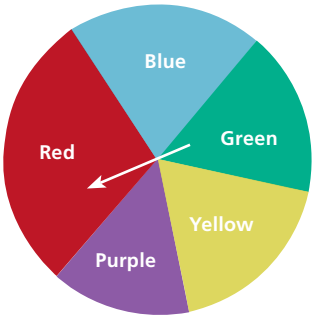
ii) What is the relative frequency for travelling by train or by ferry?

_____

## QUESTION 2

**PREDICT THE PROBABILITY OF AN EVENT FROM EXPERIMENTAL DATA USING RELATIVE FREQUENCIES**

The spinner shown below was spun 150 times and the results are shown in the table.




| OUTCOME | FREQUENCY |
|---|---|
| Blue | 28 |
| Green | 23 |
| Yellow | 24 |
| Purple | 21 |
| Red | 54 |

If the spinner was spun 500 times, how many times would you expect it to come up red? _____

_____

## QUESTION 3

**CALCULATE PROBABILITIES FROM VENN DIAGRAMS AND TWO-WAY TABLES**

a) The Venn diagram shows the sports that are played in year 9.

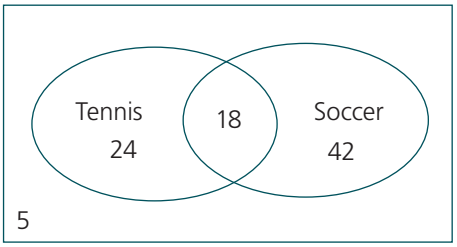
i) How many students are there altogether? _____

ii) How many students do not play sport? _____

iii) What is the probability of choosing a student who plays both tennis and soccer? _____

iv) What is the probability of choosing a student who plays tennis or soccer? _____

Sports played in year 9



b) The two way table below shows the employment status for Luke Town by gender.

| | EMPLOYED STATUS | | |
|---|---|---|---|
| | **EMPLOYED** | **UNEMPLOYED** | |
| **FEMALE** | 320 | 54 | 374 |
| **MALE** | 430 | 36 | 466 |
| | 750 | 90 | 840 |

i) How many females were unemployed? _____

ii) What is the probability of choosing a male or female who is unemployed?

_____

iii) What is the probability of randomly choosing someone who is male and employed?

_____

# MATHEMATICS STAGE 5.2

# Mathematics Stage 5.2: Financial Mathematics

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### NUMBER AND ALGEBRA: FINANCIAL MATHEMATICS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve problems involving earning money and taxation | |
| ☐ Apply the simple interest formula to solve problems, including buying on terms | |
| ☐ Calculate compound interest using repeated applications of simple interest | |

## STAGE 5.2: FINANCIAL MATHEMATICS

### SYLLABUS REFERENCE

**MA5.2-4NA** - A student solves financial problems involving compound interest.

| QUESTION 1 | APPLY THE COMPOUND INTEREST FORMULA TO SOLVE PROBLEMS |
|---|---|

(a) i) Use the compound interest formula to determine the value of an investment of $5000 compounded yearly at 4% p.a. at the end of 3 years.

ii) Calculate the amount of interest gained on the investment.

b) Using the information below:

| TYPE A | TYPE B |
|---|---|
| $10 000 compounded yearly at 5% p.a for 5 years | $10 000 compounded monthly at 5% p.a for 5 years |

determine which is the best investment. (*show working*)

c) An investment is worth $10 000 at the end of 3 years compounded annually. Calculate the interest rate that was applied if the initial principal amount was $6000. (*answer to 1 decimal place*)

d) Lisa received a bonus of $7500 at the end of the year. She needs a total of $10 000 to go on her dream holiday. Use the guess and refine method to determine how long it will take to reach her goal if she invests her bonus at 7% p.a. compounded yearly.

**QUESTION 2** **SOLVE PROBLEMS INVOLVING DEPRECIATION**

a) A computer depreciates in value at 15% p.a. If the computer was bought for $2300, how much is it worth after 3 years?

b) A Toyota Corolla was bought for $23 000 and depreciates at a rate of 21% p.a.

i) Calculate the value of the car at the end of 3 years?

ii) How much did the car depreciate in value?

# Mathematics Stage 5.2: Rates and Ratios

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: RATES AND RATIOS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Simplify ratios | |
| ☐ Solve problems involving ratios | |
| ☐ Convert into simplified rate | |
| ☐ Solve problems involving rates | |
| ☐ Interpret and draw distance/time graphs | |

## STAGE 5.2: RATES AND RATIOS

### SYLLABUS REFERENCE

**MA5.2-5NA** -A student recognises direct and indirect proportion, and solves problems involving direct proportion.

| QUESTION 1 | CONVERT BETWEEN UNITS OF RATES |
|---|---|

Convert the following:

a) \$5.50/h to \$/min _____

b) 250 L/h to mL/h _____

c) 65 km/h to m/s _____

**QUESTION 2** **SOLVE PROBLEMS INVOLVING CONSTANT RATES OF CHANGE**

a) Calculate the average speed of a car that travels 250 km in 5 hours.

b) A tap that is leaking has filled a 2.5 L bucket in 6 hours. At what rate was the tap leaking per hour (in mL/h)?

c) A truck used 45 L of petrol in 300 km. Express the fuel consumption in litres per 100 km.

**QUESTION 3** **IDENTIFY EXAMPLES OF DIRECT AND INVERSE PROPORTION**

Classify the following as inverse (indirect) or direct proportion.

a) As the number of hours at work increases, the earnings also increase.

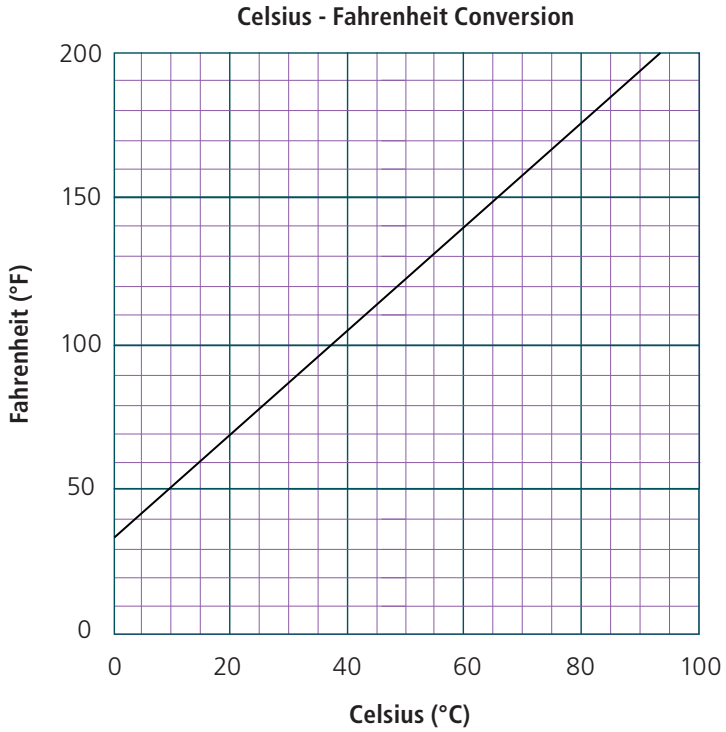
b) As speed increases, the time taken to travel a particular distance decreases.

c) As the size of the airplane increases, the cost to construct the airplane increases.

**QUESTION 4**

**INTERPRET GRAPHS OF CHANGE WHERE THE RELATIONSHIP BETWEEN THE VARIABLES IS CONSTANT**

Interpret this temperature conversion graph and answer the following questions:




a) convert 20°C to °F. _____

b) convert 100°C to °F. _____

c) is the gradient of the graph positive or negative? _____

d) are the variables in direct linear proportion? Explain your answer.

_____

_____

**QUESTION 5** **CALCULATE THE CONSTANT OF PROPORTIONALITY**

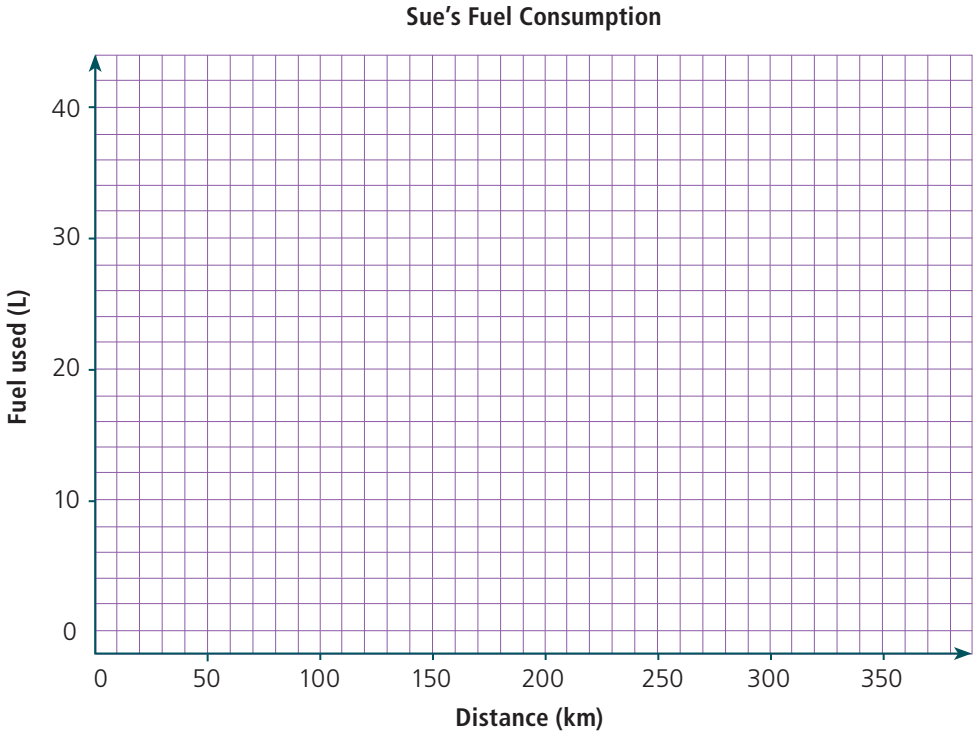
Sue's fuel usage ($F$) is in direct linear proportion with the distance that she travels ($d$).

a) Establish an equation that can be used to calculate the amount of fuel she uses on her trips.

b) Calculate the constant of proportionality if she used 25 L while travelling 200 km.

c) What distance could she travel on 40 L of fuel?

d) Construct a graph to model Sue's fuel usage.



# Mathematics Stage 5.2: Algebraic Techniques

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: ALGEBRAIC TECHNIQUES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Use letters to represent numbers | |
| ☐ Recognise and use simple equivalent algebraic expressions | |
| ☐ Simplify algebraic expressions involving the four operations | |
| ☐ Substitute into algebraic expressions | |
| ☐ Expand and factorise simple algebraic expressions | |

## STAGE 5.2: ALGEBRAIC TECHNIQUES

### SYLLABUS REFERENCE

**MA5.2-6NA** - A student simplifies algebraic fractions, expands and factorises quadratic expressions.

### QUESTION 1 — APPLY THE FOUR OPERATIONS TO SIMPLIFY SIMPLE ALGEBRAIC FRACTIONS

a) Simplify the following:

i) $\frac{x}{2} + \frac{x}{3}$

ii) $\frac{4y}{5} - \frac{2y}{3}$

iii) $\frac{2a}{3} \times \frac{3a}{8}$

iv) $\frac{xy}{5} \div \frac{y}{15}$

v) $\frac{6}{n} - \frac{1}{2n}$

b) Expand and simplify:

i) 2y(y – 3) + 3(2y + 4)

ii) 4a(10a – 7) – (3a – 4)

iii) (x + 2)(x + 4)

iv) (k – 3)(2k + 5)

**QUESTION 2**

**FACTORISE ALGEBRAIC EXPRESSIONS INCLUDING MONIC QUADRATIC TRINOMIALS**

a) Factorise the following:

i) $4d^3 - 4d$

ii) $21xy - 3x + 9x^2$

iii) $h^2 + 8h + 12$

# Mathematics Stage 5.2: Indices

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### NUMBER AND ALGEBRA: INDICES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Apply the index laws to simplify algebraic expressions with positive-integer indices and the zero index | |
| ☐ Define and use negative-integer indices to evaluate numerical expressions | |

## STAGE 5.2: INDICES

### SYLLABUS REFERENCE

**MA5.2-7NA** - A student applies index laws to operate with algebraic expressions involving integer indices.

### QUESTION 1

**WRITE EXPRESSIONS INVOLVING NEGATIVE INDICES AS EXPRESSIONS INVOLVING POSITIVE INDICES, AND VICE VERSA**

a) Write the following with a negative index:

i) $\frac{1}{x^2}$ _____

ii) $\frac{2}{m^3}$ _____

iii) $\frac{1}{4a^2}$ _____

b) Write the following with positive indices:

i) $x^{-3}$ _____

ii) $4m^{-5}$ _____

c) By substituting $x = 1$, show whether $x^{-3}$ is equivalent to $-3x$.

_____

_____

_____

## QUESTION 2

**SIMPLIFY ALGEBRAIC PRODUCTS AND QUOTIENTS INVOLVING NEGATIVE INDICES**

a) Simplify the following:

i) $2m^{-6} \times 3m^{-4}$ _____

ii) $(x^5)^{-2}$ _____

iii) $3y^{-4} \times y^7$ _____

iv) $15k^7 \div 5k^{-3}$ _____

v) $(4m^{-2})^3$ _____

b) State whether the following are true or false:

i) $4x^0 = 1$ _____

ii) $a^6 \div a^8 = a^2$ _____

iii) $(6m)^0 = 1$ _____

iv) $3x^6 \div 3x^6 = x$ _____

v) $4c^{-3} = \frac{1}{4c^3}$ _____

vi) $12m^{10} \times 2m^{-2} = 24m^8$ _____

## QUESTION 3

**SIMPLIFY ALGEBRAIC EXPRESSIONS AND FRACTIONS, INCLUDING THOSE INVOLVING INDICES**

Simplify the following:

a) $-3m^2(5m^2 + 2m^4n)$

b) $\frac{16x^2}{4x}$

c) $\frac{20a^3b^2}{15ab^3}$

d) $\frac{3xy}{2} \times \frac{2x}{6}$

e) $\frac{b^2c^3}{4} \times \frac{12}{bc^2}$

f) $\frac{4n^3}{3m^2} \div \frac{12n}{3m^5}$

# Mathematics Stage 5.2: Equations

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: EQUATIONS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve simple linear equations using algebraic techniques | |
| ☐ Solve simple quadratic equations of the form $x^2 = c$ | |

## STAGE 5.2: EQUATIONS

### SYLLABUS REFERENCE

**MA5.2-8NA** - A student solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques.

### QUESTION 1 — SOLVE LINEAR EQUATIONS INVOLVING GROUPING SYMBOLS

Solve the following:

a) $2(y + 4) + 3(y - 2) = 10$

b) $3(2y + 5) = 4y - 9$

c) $4(2k - 6) = -3(2k + 3)$

**QUESTION 2** **SOLVE LINEAR EQUATIONS INVOLVING ALGEBRAIC FRACTIONS**

Solve the following:

a) $\frac{3y + 4}{3} = 5$

b) $\frac{y - 2}{4} + 5 = 7$

c) $\frac{3x}{5} - 4 = -2$

d) $\frac{2m}{2} + \frac{m}{3} = 4$

e) $\frac{7y + 2}{4} = \frac{2y + 5}{2}$

## QUESTION 3

**SOLVE QUADRATIC EQUATIONS OF THE FORM $AX^2 = C$**

a) Solve the following, leave answers to 1 decimal place where necessary:

i) $y^2 = 4$ _____

ii) $2m^2 = 32$ _____

iii) $-5x^2 = -40$ _____

b) Solve the following, leaving answers in exact form:

i) $k^2 = 20$ _____

ii) $6k^2 = 4$ _____

## QUESTION 4

**SOLVE QUADRATIC EQUATIONS OF THE FORM $AX^2 + BX + C = 0$ (WHERE A = 1) USING FACTORS**

Solve the following quadratic equations:

i) $x^2 + 2x + 1 = 0$ _____

ii) $y^2 - 5y + 6 = 0$ _____

iii) $z^2 - 9z - 10 = 0$ _____

## QUESTION 5

**SOLVE EQUATIONS RESULTING FROM SUBSTITUTION INTO FORMULAS**

a) The perimeter of a rectangle is given by $P = 2L + 2B$, find:

i) B: if $L = 5$ and $P = 25$

ii) $L$: if $P = 40$ and $B = 4$

b) The formula for converting temperature recorded in °F to temperature in °C, is $C = \frac{5}{9}(F - 32)$. Express 80°F in °C, correct to the nearest degree.

**QUESTION 6** **SOLVE WORD PROBLEMS USING LINEAR EQUATIONS**

a) The sum of three consecutive integers is 42. What are the three integers?

b) The area of a triangle with base 10 cm is 45 $cm^2$. Find the perpendicular height of the triangle.

## QUESTION 7

**SOLVE LINEAR INEQUALITIES AND GRAPH THE SOLUTIONS**

Solve the following inequalities and then graph the solutions on a number line.

a) $2x - 1 > 9$

b) $10 < b - 5$

c) $3(b - 2) \geq 18$

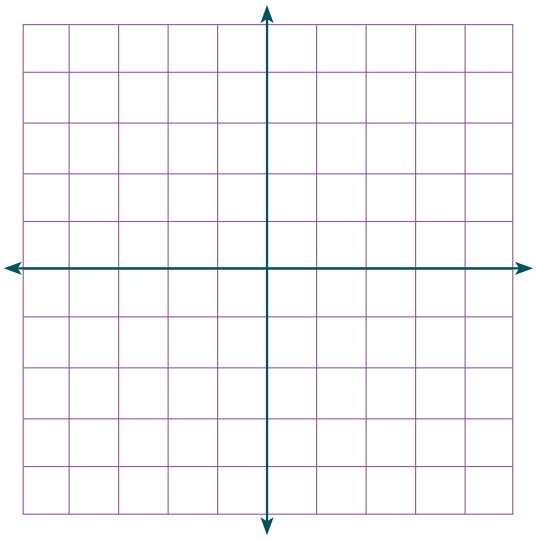
d) $\frac{y + 5}{3} < -2$

e) $2 - 5m \leq 6$

## QUESTION 8

SOLVE LINEAR SIMULTANEOUS EQUATIONS USING ALGEBRAIC AND GRAPHICAL TECHNIQUES

a) On the same set of axis, graph $y = 9 - 2x$ and $y = 4x - 6$, then solve simultaneously.



b) Solve the following equations simultaneously using the substitution method.

$$\left\{ \begin{array}{l} y = x + 4 \\ y = 3x - 2 \end{array} \right\}$$

c) Solve the following equations simultaneously using the elimination method.

$$\left\{ \begin{array}{l} x + 3y = 7 \\ 4x - 3y = 13 \end{array} \right\}$$

# Mathematics Stage 5.2: Linear Relationships

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### NUMBER AND ALGEBRA: LINEAR RELATIONSHIPS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Find the midpoint, gradient and length of intervals on the Cartesian plane using informal strategies | |
| ☐ Graph linear relationships from equations | |
| ☐ Determine that parallel lines on the Cartesian plane have equal gradients | |

## STAGE 5.2: LINEAR RELATIONSHIPS

### SYLLABUS REFERENCE

**MA5.2-9NA** - A student uses the gradient-intercept form to interpret and graph linear relationships.

### QUESTION 1

APPLY THE GRADIENT-INTERCEPT FORM OF THE EQUATION OF A STRAIGHT LINE TO INTERPRET AND GRAPH STRAIGHT LINES

a) Find the gradient and y intercept of the following linear equations:

| Equation | Gradient | Y intercept |
|---|---|---|
| i) $y = 3x - 4$ | | |
| ii) $y = 5 - 4x$ | | |
| iii) $6y - 3x = 6$ | | |
| iv) $2x - 2y + 3 = 0$ | | |

b) Write the equation of a straight line with gradient $m = -2$ and $y$ intercept of 6.

_____

c) Graph the following line by first determining the gradient and y intercept of the equation:

$y = 2 - 4x$

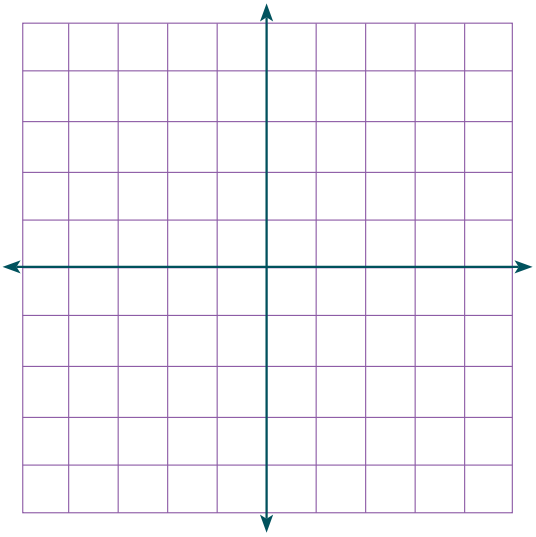
_____

_____

_____

_____

_____



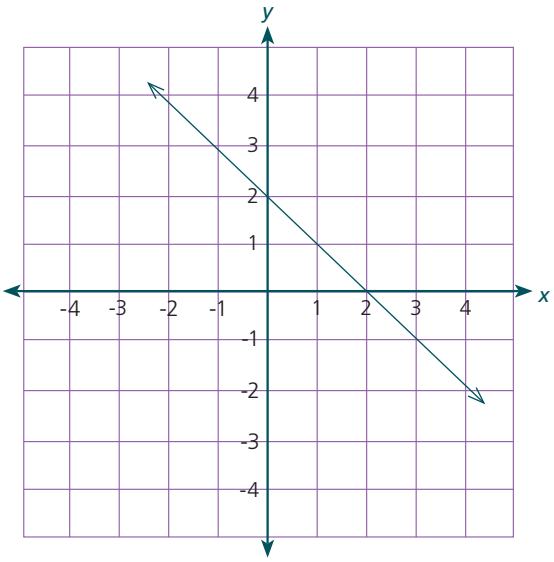
d) Determine the following for the given line:

i) the y intercept. _____

ii) the gradient. _____

iii) the equation of the line.

_____



## QUESTION 2

**APPLY THE PROPERTIES OF THE GRADIENTS OF PARALLEL AND PERPENDICULAR LINES ON THE CARTESIAN PLANE**

a) Circle which of the following lines are perpendicular to each other.

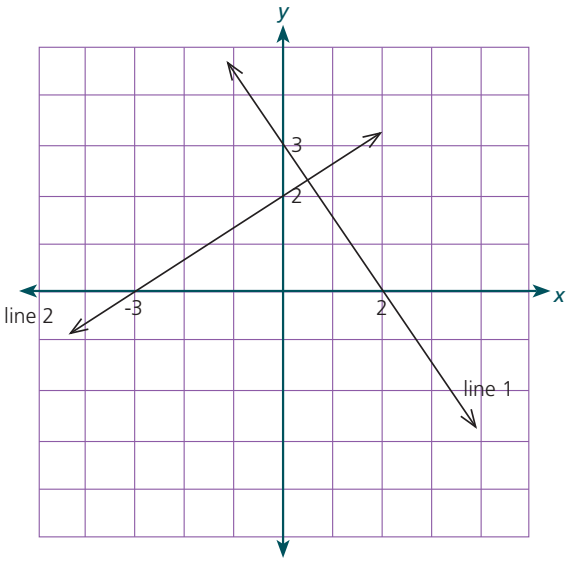
(i) $y = -2x + 2$ (ii) $y = -3x + 2$ (iii) $y = \frac{1}{2}x + 4$ (iv) $y = -\frac{1}{2}x + 4$

b) Determine the equation of a line with y-intercept at -2 that is:

i) parallel to $y = 3 - 4x$.

ii) perpendicular to the line $2x - 3y + 4 = 0$.

c) Determine the equation of the following two lines and show that they are perpendicular.



# Mathematics Stage 5.2: Non-Linear Relationships

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

**NUMBER AND ALGEBRA: NON-LINEAR RELATIONSHIPS**

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Graph simple parabolas, exponentials and circles on the Cartesian plane using tables of values and digital technologies | |

## STAGE 5.2: NON-LINEAR RELATIONSHIPS

**SYLLABUS REFERENCE**

**MA5.2-10NA** - A student connects algebraic and graphical representations of simple non-linear relationships.

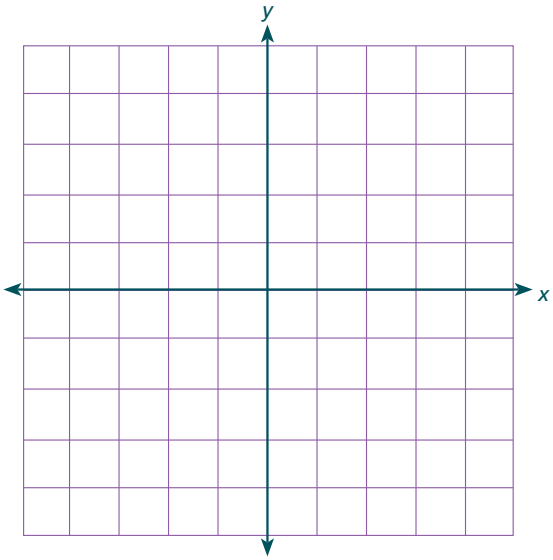
**QUESTION 1**

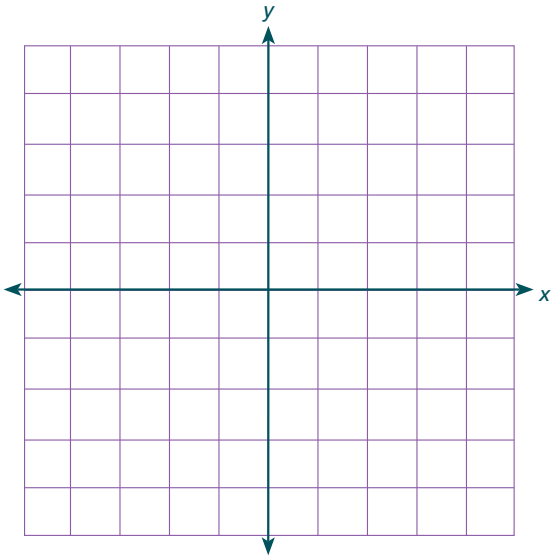
**IDENTIFY, DRAW AND COMPARE GRAPHS OF PARABOLAS OF THE FORM $Y = AX^2 + C$**

a) Graph and label the following sets of parabolas on the same Cartesian plane.

i) $y = x^2$ $y = -x^2$ $y = 2x^2$

ii) $y = x^2$ $y = x^2 + 2$ $y = x^2 - 1$



iii) Comment on any similarities or differences between the graphs including the co-efficient of the $x^2$ and the constant term.

**DETERMINE THE EQUATION OF A PARABOLA**

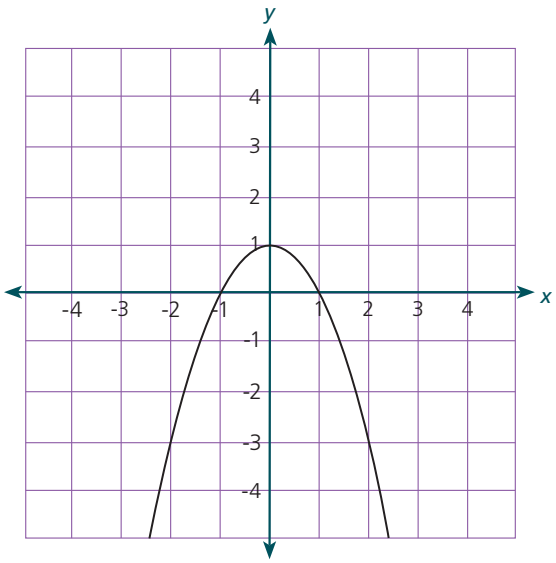
b) Find the equation of a parabola with vertex at (0,-1).

**DETERMINE THE X-COORDINATE OF A POINT ON A PARABOLA, GIVEN THE Y-COORDINATE OF THE POINT**

c) Find the x coordinate of the parabola $y = 2x^2 - 7$ whose $y$ coordinate is 5.

**DETERMINE THE EQUATION OF A PARABOLA**

d) Determine the equation of the parabola below.

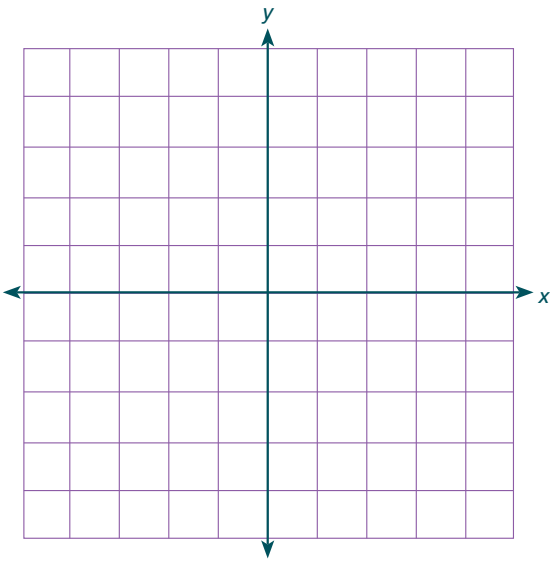


## QUESTION 2

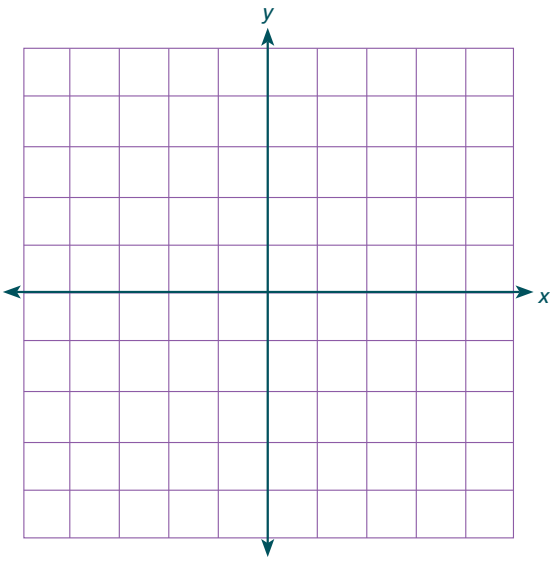
**SKETCH, COMPARE AND DESCRIBE THE KEY FEATURES OF SIMPLE EXPONENTIAL CURVES**

Sketch and describe the similarities and differences of the graphs.

a) $y = 2^x$ $y = -2^x$ $y = 2^{-x}$




b) $y = 2^x$ $y = 2^x + 1$ $y = 2^x - 1$



## QUESTION 3

RECOGNISE AND DESCRIBE EQUATIONS THAT REPRESENT CIRCLES

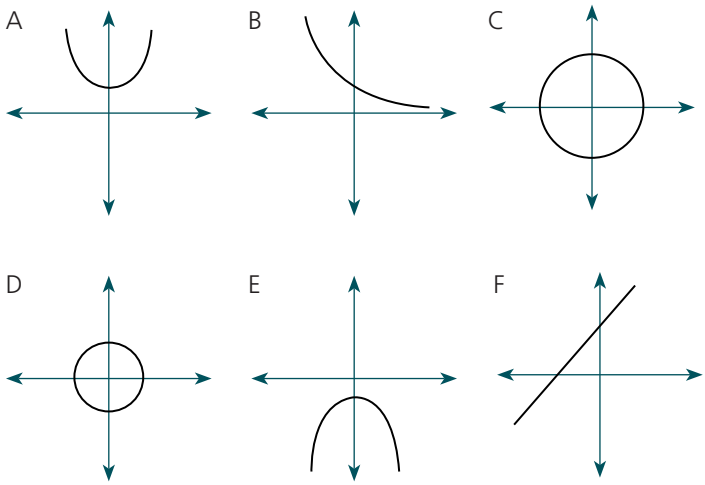
a) Find the centre and radius of a circle $x^2 + y^2 = 9$.

b) Write the equation of a circle with radius (0,0) and radius 2 units.

## QUESTION 4

MATCH GRAPHS OF STRAIGHT LINES, PARABOLAS, CIRCLES AND EXPONENTIALS TO THE APPROPRIATE EQUATIONS

a) Match the following graphs to their equation.




i) $y = 2^{-x}$ _____

ii) $25 = x^2 + y^2$ _____

iii) $y = -x^2 - 1$ _____

iv) $y = x^2 + 3$ _____

v) $x^2 + y^2 = 9$ _____

vi) $y = x + 4$ _____

# Mathematics Stage 5.2: Area and Surface Area

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### MEASUREMENT AND GEOMETRY: AREA AND SURFACE AREA

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve problems involving areas of composite shapes by dissection into triangles, quadrilaterals, quadrants, semicircles and sectors | |
| ☐ Solve problems involving the surface area of rectangular and triangular prisms | |

## STAGE 5.2: AREA AND SURFACE AREA

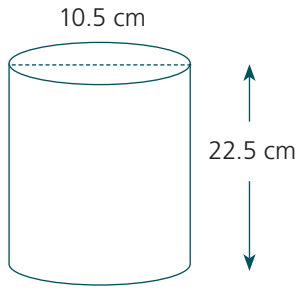
### SYLLABUS REFERENCE

**MA5.2-11MG** - A student calculates the surface areas of right prisms, cylinders and related composite solids.

### QUESTION 1 — CALCULATE THE SURFACE AREAS OF CYLINDERS

Calculate the surface area of the following:

a)

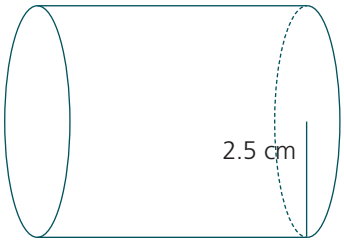



(closed cylinder)

b)


(closed half cylinder)

c)



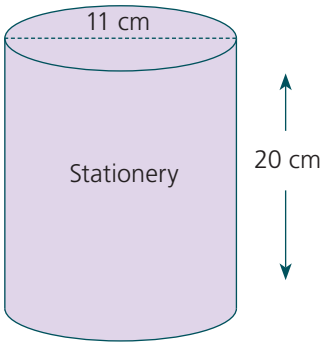

78.6 mm

(cylinder open at one end)

**QUESTION 2**

**SOLVE A VARIETY OF PRACTICAL PROBLEMS RELATED TO SURFACE AREAS OF PRISMS, CYLINDERS AND RELATED COMPOSITE SOLIDS**

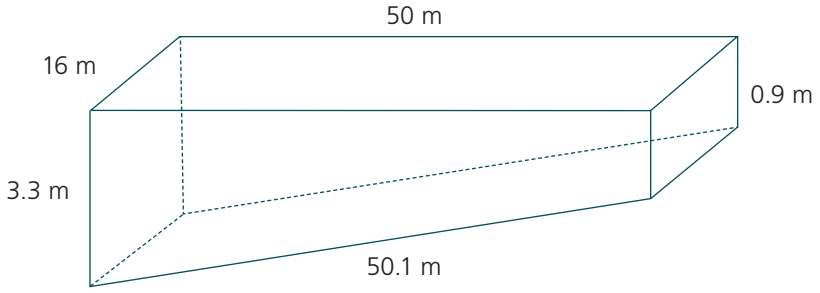
a) The following open cylindrical tin is empty and will be used to store stationery.



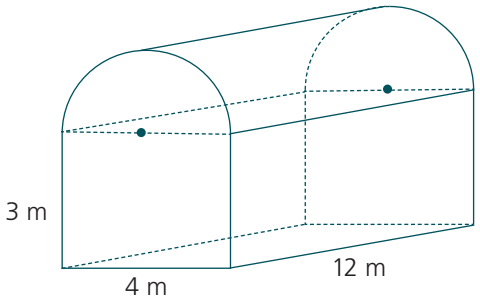

i) Calculate the surface area of the label around the tin.

ii) Calculate the surface area of the tin.

b) A swimming pool has the shape of a trapezoidal prism as shown in the diagram below.
If this pool is to be tiled, find the cost of tiling the pool if tiles cost $45 per square metre.




c) The curved roof of a green house is to be covered in shade cloth.




i) Calculate, correct to 1 decimal place, the area of shade cloth needed if there are no overlaps.

ii) Shade cloth is sold in 1.6 m wide rolls. How many linear metres of shade cloth are needed to cover the curved roof? Answer to the nearest 0.1 metres.

# Mathematics Stage 5.2: Volume

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### MEASUREMENT AND GEOMETRY: VOLUME

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Visualise and draw different views of three dimensional objects | |
| ☐ Convert between metric units of volume and capacity | |
| ☐ Establish and use formulas to find volumes of right prisms and cylinders | |
| ☐ Solve problems involving volume and capacity | |

## STAGE 5.2: VOLUME

### SYLLABUS REFERENCE

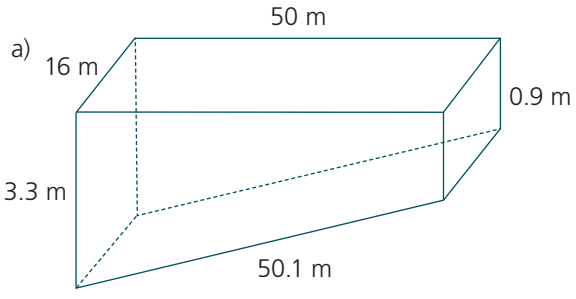
**MA5.2-12MG** - A student applies formulas to calculate the volume of composite solids composed of right prisms and cylinders.

### QUESTION 1

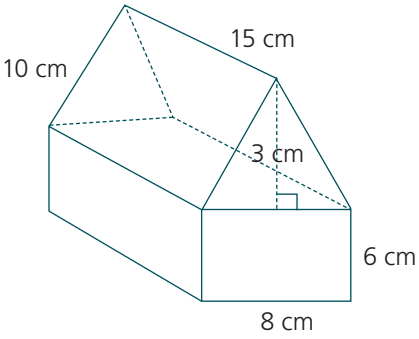
**FIND THE VOLUMES OF COMPOSITE SOLIDS**

Calculate the volume area of the following:

a)

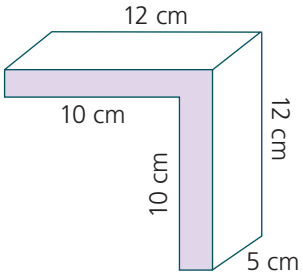



b)



c)

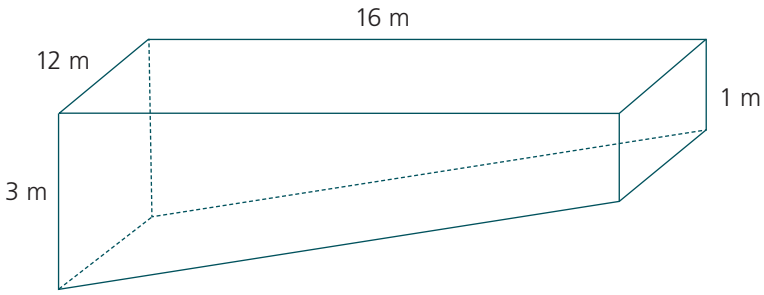

d)




**QUESTION 2**

**SOLVE A VARIETY OF PRACTICAL PROBLEMS RELATED TO THE VOLUMES AND CAPACITIES OF PRISMS**

The swimming pool below is 16 m long and 12 m wide. The depth of the water ranges from 1 m to 3 m.




Calculate the capacity of the pool in kilolitres.

**QUESTION 3**

**SOLVE A VARIETY OF PRACTICAL PROBLEMS RELATED TO THE VOLUMES AND CAPACITIES OF PRISMS**

Rice crackers of diameter 38 mm are packed in a cardboard box of height 22 cm.
Calculate correct to 2 decimal places:

a) the volume of the crackers in the box.


b) the volume of the box.

c) the percentage of the box that is empty space.

# Mathematics Stage 5.2: Right-Angled Triangles (Trigonometry)

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### MEASUREMENT AND GEOMETRY: TRIGONOMETRY

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Use trigonometry to find sides and angles in right-angled triangles (angles measured to nearest degree) | |
| ☐ Apply trigonometry to solve problems involving right-angled triangles, including the use of angles of elevation and depression, from a given diagram | |

## STAGE 5.2: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

### SYLLABUS REFERENCE

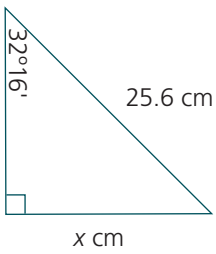
**MA5.2-13MG** - A student applies trigonometry to solve problems, including problems involving bearings.

### QUESTION 1

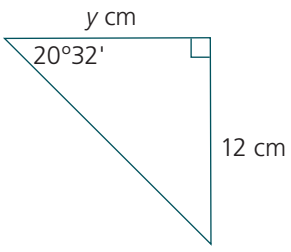
**USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (INCLUDING ANGLES MEASURED TO NEAREST MINUTE)**

Find the pronumeral in the following to 2 decimal places:

a)




b)

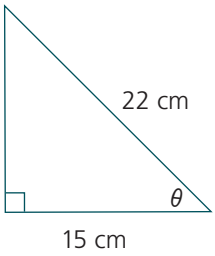


## QUESTION 2

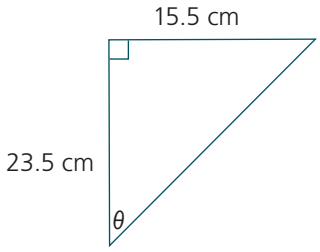
**USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (INCLUDING ANGLES MEASURED TO NEAREST MINUTE)**

Find the following angles to the nearest minute:

a)




b)



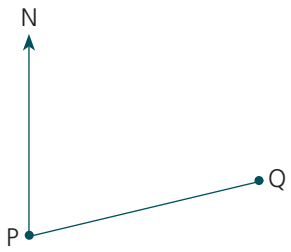

## QUESTION 3

**APPLY TRIGONOMETRY TO SOLVE WORD PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES, INCLUDING THE USE OF THREE-FIGURE BEARINGS AND ANGLES OF ELEVATION AND DEPRESSION**

a) A 15 m ladder is leaning against a wall. The angle the ladder makes with the ground is 28°15'.
How far is the foot of the ladder from the wall?

b) The angle of depression from the top of a cliff is 58°. A boat out at sea is 125 m from the base of the cliff.
Calculate the height of the cliff to the nearest metre.

c) The bearing of Q from P in the diagram below is 54°.




Calculate the bearing of P from Q.

d) John starts walking from his camping site to collect wood. He walks 350 m north and then 230 m west and then he stops.

i) Draw a diagram of John's path.

ii) How far is he away from camp to the nearest metre?

iii) Calculate his bearing from camp to the nearest minute.

# Mathematics Stage 5.2: Properties of Geometric Figures

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### MEASUREMENT AND GEOMETRY: PROPERTIES OF GEOMETRIC FIGURES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Identify similar figures and describe their properties informally | |
| ☐ Use scale factors and proportion statements to determine unknown lengths in similar figures | |
| ☐ Use scales on maps, plans and drawings | |

## STAGE 5.2: PROPERTIES OF GEOMETRIC FIGURES

### SYLLABUS REFERENCE

**MA5.2-14MG** - A student calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar.

| QUESTION 1 | WRITE FORMAL PROOFS OF THE CONGRUENCE OF TRIANGLES |
|---|---|

a) State the four tests for congruent triangles.

_____

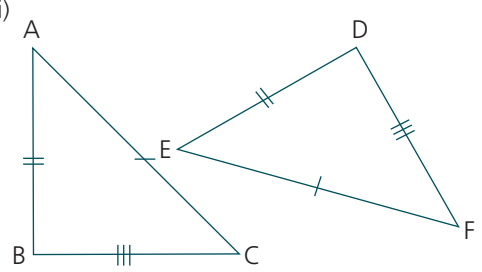
_____

_____

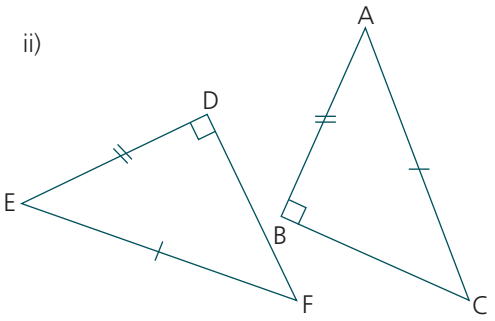
_____

b) Are the following triangles congruent? State the test used to justify that the triangles are congruent.

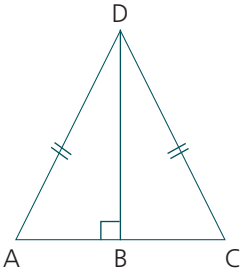
i)




ii)




c) In the diagram below:



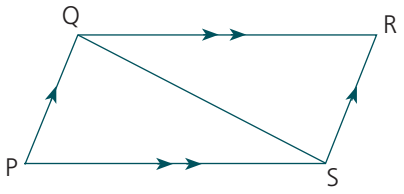

i) prove that $\Delta ABD \equiv \Delta CBD$

ii) hence, prove that $\angle DAB = \angle DCB$

## QUESTION 2

**USE THE CONGRUENCE OF TRIANGLES TO PROVE PROPERTIES OF THE SPECIAL QUADRILATERALS**

In the diagram below:




i) prove that $\Delta PQS \equiv \Delta RSQ$

ii) hence, prove that PQRS is a parallelogram

## QUESTION 3

**DETERMINE THE MINIMUM CONDITIONS FOR TRIANGLES TO BE SIMILAR**

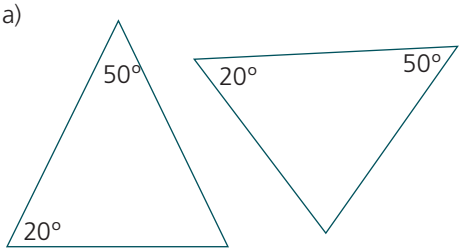
List the tests to justify if two triangles are similar.

## QUESTION 4

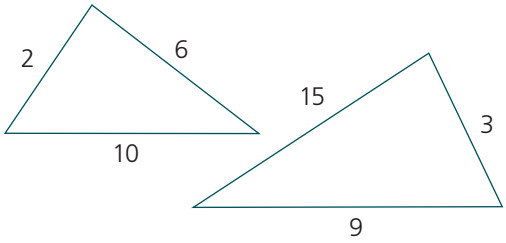
**DETERMINE WHETHER TWO TRIANGLES ARE SIMILAR USING AN APPROPRIATE TEST**

State the test that proves the triangles below are similar.

a)



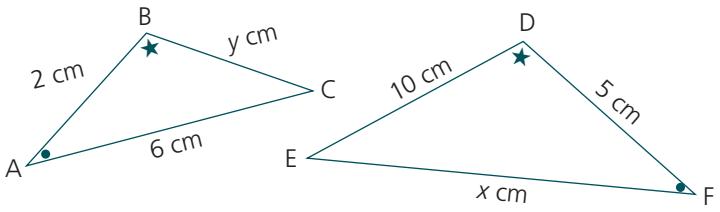

b)




## QUESTION 5

**APPLY LOGICAL REASONING, INCLUDING THE USE OF SIMILARITY TO NUMERICAL EXERCISES INVOLVING PLANE SHAPES**

Find the value of the pronumeral in the following:



**QUESTION 6**

**ESTABLISH AND USE THE SUM OF EXTERIOR ANGLES AND THE SUM OF INTERIOR ANGLES RESULTS FOR POLYGONS**

A regular convex polygon has 20 sides, find:

i) the sum of its interior angles

ii) the size of each interior angle

iii) the size of each exterior angle

# Mathematics Stage 5.2: Single Variable Data Analysis

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### STATISTICS AND PROBABILITY: SINGLE VARIABLE DATA ANALYSIS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Construct and interpret back-to-back stem and leaf plots | |
| ☐ Describe data, using terms including 'skewed', 'symmetric' and 'bi-modal' | |
| ☐ Compare two sets of numerical data in a display using mean, median and range | |
| ☐ Interpret and critically evaluate reports in the media and elsewhere that link claims to data displays and statistics | |

## STAGE 5.2: SINGLE VARIABLE DATA ANALYSIS

### SYLLABUS REFERENCE

**MA5.2-15SP** - A student uses quartiles and box plots to compare sets of data, and evaluates sources of data.

### QUESTION 1 — DETERMINE QUARTILES AND INTERQUARTILE RANGE

For the following set of data:

{2 3 4 5 5 6 7 7 8 12}

a) determine the 'five number summary' {upper and lower extremes, median, upper and lower quartile}.

_____

_____

b) determine the interquartile range for this set of data.

_____

_____

CONSTRUCT, INTERPRET AND USE BOX PLOTS TO COMPARE SETS OF DATA

c) What percentage of data is contained within the interquartile range?

d) Construct a box plot for this data set?

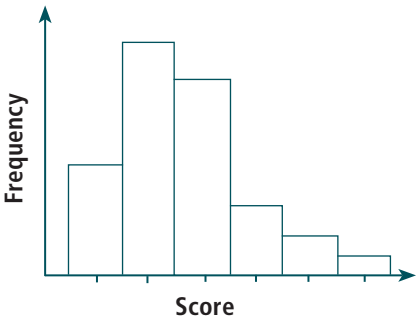
e) What percentage of data is less than 4?

**QUESTION 2**

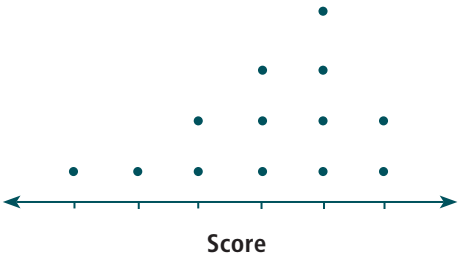
COMPARE SHAPES OF BOX PLOTS TO CORRESPONDING HISTOGRAMS AND DOT PLOTS

Draw a line and match the following histogram and dot plot to their corresponding box plots.

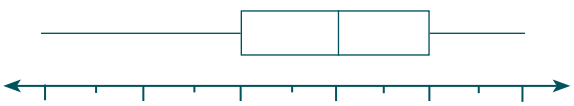
a)



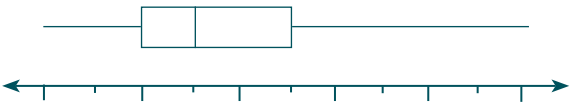

b)




i)



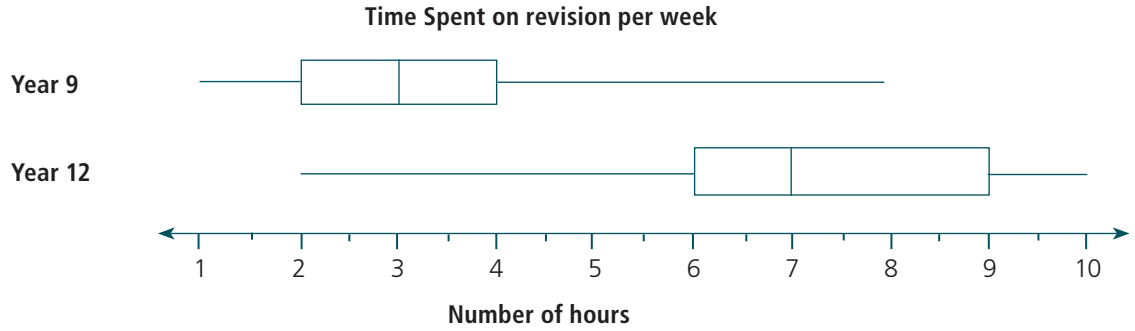
ii)



**QUESTION 3**

**DESCRIBE SIMILARITIES AND DIFFERENCES BETWEEN TWO SETS OF DATA DISPLAYED IN PARALLEL BOX PLOTS**

Year 9 and year 12 students were surveyed on how much time they spent on revision per week. The data collected has been displayed in the parallel box plots below:




a) compare the range for year 9 and year 12.

b) calculate the interquartile range for both data sets.

c) find the median time spent on revision for the year 9 and year 12 students.

d) which class spent the longest time on revision? Comment on why this may be the case.

e) what percentage of students studied more than 7 hours in year 12?

# Mathematics Stage 5.2: Bivariate Data Analysis

Name: _____

Class: _____

## STAGE 5.2: BIVARIATE VARIABLE DATA ANALYSIS

### SYLLABUS REFERENCE

**MA5.2-16SP** - A student investigates relationships between two statistical variables, including their relationship over time.

### QUESTION 1

**RECOGNISE THE DIFFERENCE BETWEEN AN INDEPENDENT VARIABLE AND ITS DEPENDENT VARIABLE**

Consider the following statements and identify the dependent variable and the independent variable in each:

a) The amount of rainfall varies with each month of the year.

Dependent variable: _____

Independent variable: _____

b) The heights and hand-spans of a group of students.

Dependent variable: _____

Independent variable: _____

### QUESTION 2

**RECOGNISE THE DIFFERENCE BETWEEN AN INDEPENDENT VARIABLE AND ITS DEPENDENT VARIABLE**

Which type of graph is the most appropriate method of representing data collected over time, and why?

_____

_____

## QUESTION 3

**RECOGNISE THE DIFFERENCE BETWEEN AN INDEPENDENT VARIABLE AND ITS DEPENDENT VARIABLE**

The following table shows the estimated Australian rainfall for 2014 by month.

| MONTH | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RAINFALL (MM) | 115 | 41 | 45 | 28 | 22 | 20 | 17 | 18 | 5 | 10 | 37 | 82 |

a) Construct a line graph for the data above using time as the independent variable.

b) Which month was the driest?

_____

c) Which month was the wettest?

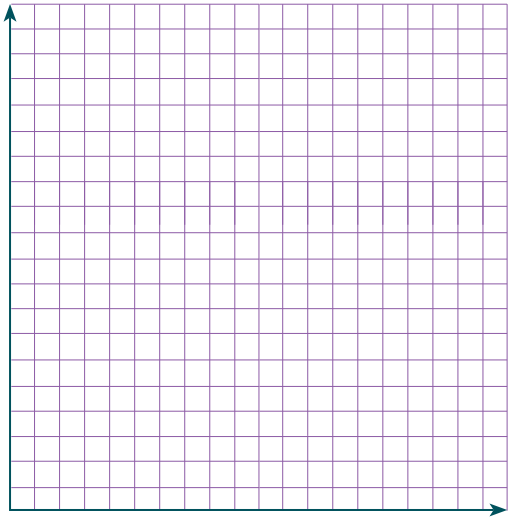
_____

d) Analyse and comment on any trends expressed in the graph.

_____

_____

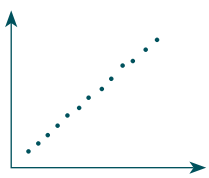
_____

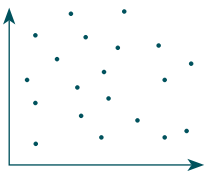


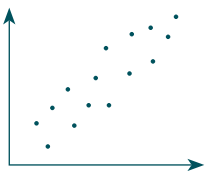
## QUESTION 4

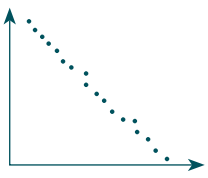
**DESCRIBE, INFORMALLY, THE STRENGTH AND DIRECTION OF THE RELATIONSHIP BETWEEN TWO VARIABLES DISPLAYED IN A SCATTER PLOT**

Classify the following as either having strong or weak, positive or negative or no correlation.







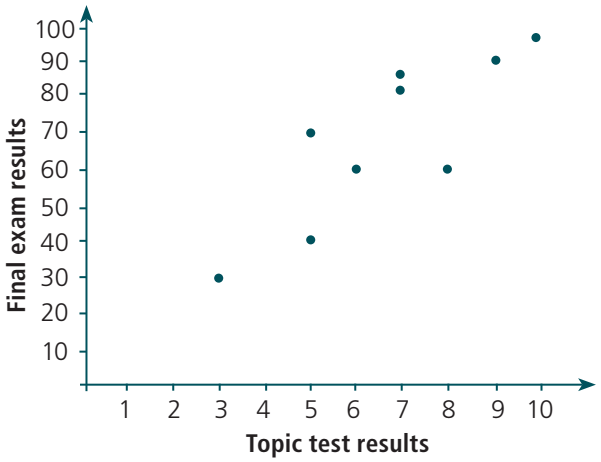


_____ _____ _____ _____

## QUESTION 5

INTERPRET SCATTER PLOTS OF TWO NUMERICAL VARIABLES

The scatter plot below shows the final exam results versus topic tests results.




a) Describe the relationship between the two variables.

b) Estimate the final exam result of a student who achieved 5 in their topic tests.

c) Analyse and comment on any trends expressed in the scatter plot.

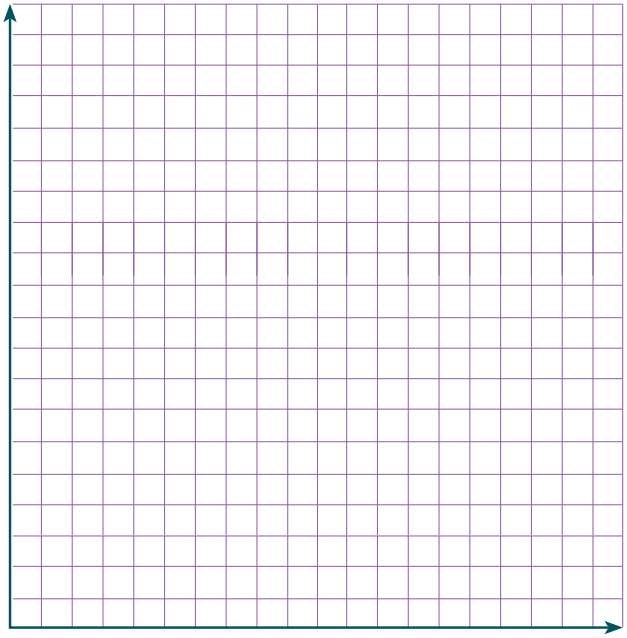
## QUESTION 6

**DESCRIBE, INFORMALLY, THE STRENGTH AND DIRECTION OF THE RELATIONSHIP BETWEEN TWO VARIABLES DISPLAYED IN A SCATTER PLOT**

John took a long journey home and recorded the distance travelled as well as the remaining fuel in the tank. The following table shows the results.

| DISTANCE TRAVELLED (KM) | 0 | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|---|---|
| FUEL IN TANK (LITRES) | 100 | 92 | 84 | 76 | 68 | 58 | 50 | 40 | 32 |

a) Construct a scatterplot for the data above using the graph below.



b) Comment on the relationship between the two variables.

# Mathematics Stage 5.2: Probability

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

### STATISTICS AND PROBABILITY: PROBABILITY

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Calculate relative frequencies to estimate probabilities | |
| ☐ Calculate probabilities from Venn diagrams and two-way tables | |

## STAGE 5.2: PROBABILITY

### SYLLABUS REFERENCE

**MA5.2-17SP** - A student describes and calculates probabilities in multistep chance experiments.

| QUESTION 1 | LIST ALL OUTCOMES FOR TWO-STEP CHANCE EXPERIMENTS AND CALCULATE PROBABILITIES FOR SIMPLE AND COMPOUND EVENTS |
|---|---|

a) A die is rolled and a coin is thrown. List the sample space.

_____

_____

b) What is the probability of obtaining a head and a 6?

_____

_____

## QUESTION 2

**CALCULATE PROBABILITIES FOR SIMPLE AND COMPOUND EVENTS**

A bag contains 4 blue counters and 3 red counters. A counter is picked out of the bag and then returned before a second counter is chosen.

a) What is the probability of choosing a blue counter on the first selection? _____

b) What is the probability of choosing a blue counter on both selections? _____

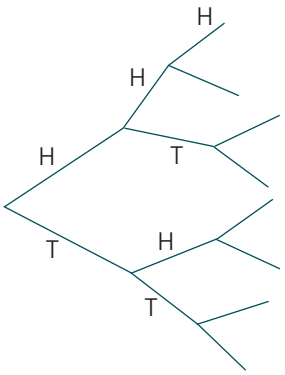
c) Calculate the probability of choosing a blue and then a red counter. _____

## QUESTION 3

**RECORD RESULTS OF TWO- AND THREE-STEP CHANCE EXPERIMENTS, AND DETERMINE PROBABILITIES OF EVENTS**

A coin is tossed three times.

a) Complete the tree diagram below and list the sample space.




HHH _____

_____

_____

_____

_____

_____

_____

_____

b) What is the probability of obtaining three heads?

_____

_____

c) What is the probability of obtaining at least one tail?

_____

_____

## QUESTION 4

**CALCULATE PROBABILITIES FOR SIMPLE AND COMPOUND EVENTS IN TWO- AND THREE-STEP CHANCE EXPERIMENTS WITH AND WITHOUT REPLACEMENT**

Jessica has a bag of jelly beans made up of 2 green, 3 red and 4 orange jelly beans, she takes a jelly bean from the bag and eats it.

a) Calculate the probability that Jessica eats a green jelly bean.

b) Calculate the probability that Jessica eats a red jelly bean next.

c) If a green and red jellybean have been eaten from the bag, calculate the probability of obtaining an orange jellybean on the third attempt.

## QUESTION 5

**DISTINGUISH BETWEEN INDEPENDENT AND DEPENDENT EVENTS INFORMALLY**

Distinguish whether the following pairs of events are dependent or independent:

a) rolling a die to obtain a 4 and rolling a second die to obtain an even number.

b) tossing a coin to obtain a head and then tossing a coin to obtain a tail.

c) a person who is short sighted and a person wearing glasses.

## QUESTION 6

**CALCULATE PROBABILITIES OF EVENTS WHERE A CONDITION IS GIVEN THAT RESTRICTS THE SAMPLE SPACE**

Given that a number greater than 2 has been rolled on a fair six sided die:

a) list the sample space.

b) calculate the probability that this number was 6.

c) calculate the probability that this number was 1.

## QUESTION 7

**CRITICALLY EVALUATE CONDITIONAL STATEMENTS IN CHANCE SITUATIONS**

a) Explain why if you toss a 1 on a six sided die, the probability of obtaining a 1 on the second toss remains the same.

b) Explain why the statement 'If you obtain a head on three consecutive tosses of a coin, then there is a greater chance of obtaining a tail on the next toss' is incorrect.

# MATHEMATICS STAGE 5.3

# Mathematics Stage 5.3: Rates and Ratios

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### NUMBER AND ALGEBRA: RATES AND RATIOS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve problems involving constant rates of change | |
| ☐ Interpret graphs of change where the relationship between the variables is constant | |
| ☐ Construct graphs of direct linear proportion | |

## STAGE 5.3: RATES AND RATIOS

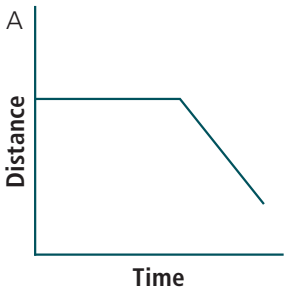
### SYLLABUS REFERENCE

**MA5.3-4NA** - A student draws, interprets and analyses graphs of physical phenomena.

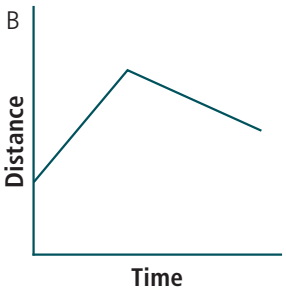
| QUESTION 1 | INTERPRET AND DESCRIBE GRAPHS OF CHANGE WHERE THE RELATIONSHIP BETWEEN THE VARIABLES IS NOT CONSTANT |
|---|---|

a) A truck driver is driving at a certain speed and then he decreases his speed, which graph represents this?

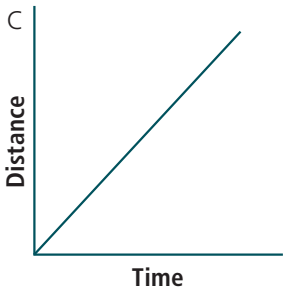
A




B




C

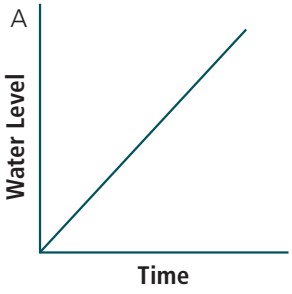


## QUESTION 1

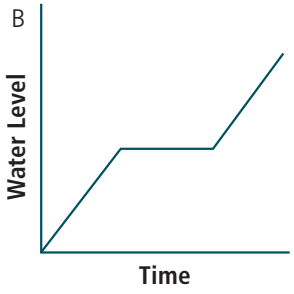
INTERPRET AND DESCRIBE GRAPHS OF CHANGE WHERE THE RELATIONSHIP BETWEEN THE VARIABLES IS NOT CONSTANT

b) A cylindrical tank is filled with water. Which graph represents this?

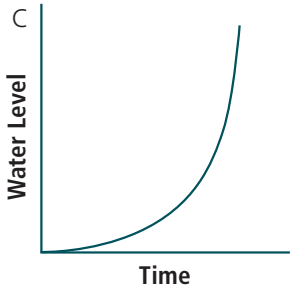
A




B



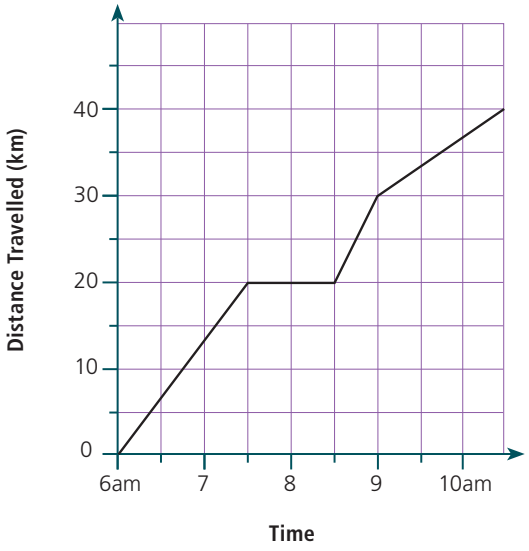

C




## QUESTION 2

INTERPRET DISTANCE/TIME GRAPHS

A cyclist left home early and went for a ride. The travel graph below shows his journey:




a) what time did his journey finish? _____

b) how long did his journey take? _____

c) what time did he rest? How long was the rest for? _____

d) between what times was he travelling the fastest? _____

e) what was the average speed of the cyclist during his journey?

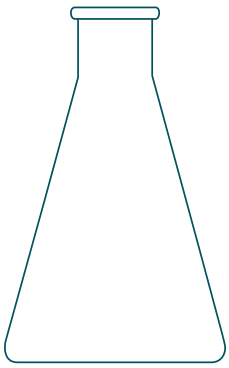
_____

_____

**QUESTION 3** **SKETCH GRAPHS OF PHYSICAL PHENOMENA**

a) A ball is dropped from the top of a 10 metre building to the ground and then it eventually stops. Draw a graph that represents the height of the ball above the ground over time.

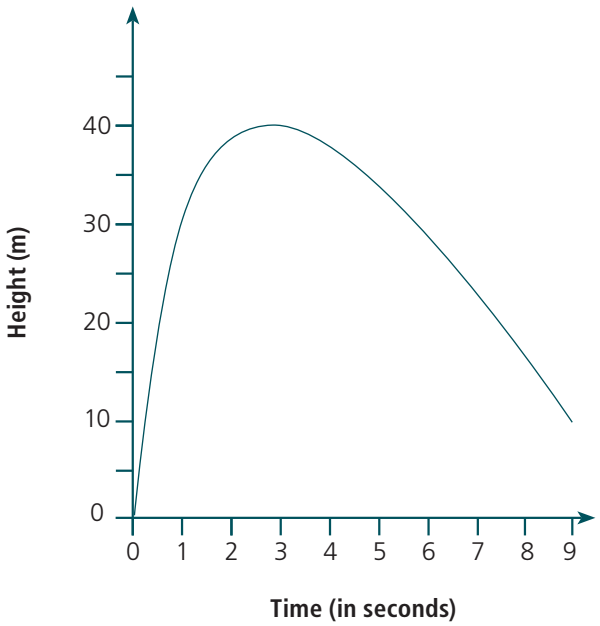
b) Water is being poured into the flask below at a constant rate. Sketch a graph that represents the level of the water in the flask over time.



## QUESTION 4

**INTERPRET GRAPHS, MAKING SENSIBLE STATEMENTS ABOUT THE RATE OF INCREASE OR DECREASE**

Jane kicks a football high up in the air and it lands on the neighbour's shed.




a) How high was the shed? _____

b) How long did it take for the ball to land on the shed? _____

c) What was the maximum height reached by the ball? _____

d) When was the height decreasing? _____

e) What was the average speed of the ball from when it was kicked in the air until it landed on the shed?

_____

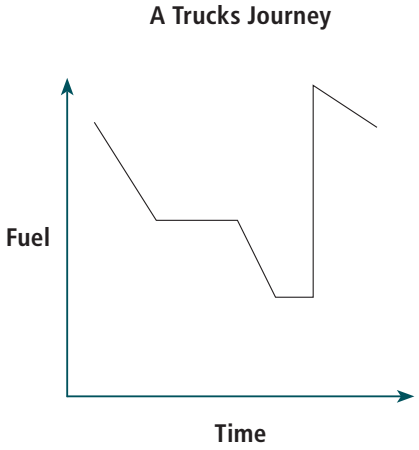
_____

_____

## QUESTION 5

**INTERPRET GRAPHS, MAKING SENSIBLE STATEMENTS ABOUT THE RATE OF INCREASE OR DECREASE**

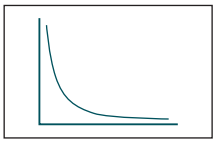
Describe what is happening in the graph below.

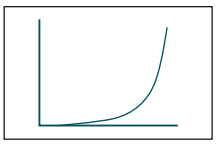


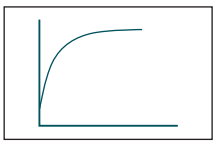

## QUESTION 6

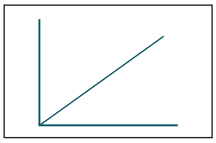
**DESCRIBE QUALITATIVELY THE RATE OF CHANGE OF A GRAPH**

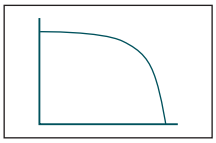
Match the graphs to their corresponding descriptions:

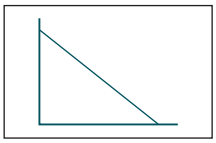












Increasing at a constant rate

Decreasing at a decreasing rate

Decreasing at a constant rate

Increasing at an increasing rate

Increasing at a decreasing rate

Decreasing at an increasing rate

# Mathematics Stage 5.3: Algebraic Techniques

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### NUMBER AND ALGEBRA: ALGEBRAIC TECHNIQUES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Apply the four operations to simplify simple algebraic fractions | |
| ☐ Expand and simplify algebraic expressions where appropriate, including binomial products | |
| ☐ Factorise monic quadratic trinomial expressions | |

## STAGE 5.3: ALGEBRAIC TECHNIQUES

### SYLLABUS REFERENCE

**MA5.3-5NA** - A student selects and applies appropriate algebraic techniques to operate with algebraic expressions.

| QUESTION 1 | ADD AND SUBTRACT ALGEBRAIC FRACTIONS WITH NUMERICAL DENOMINATORS, INCLUDING THOSE WITH BINOMIAL NUMERATORS |
|---|---|

Simplify the following:

a) $\frac{3x+2}{4} + \frac{x-7}{2}$

b) $\frac{2x}{3} - \frac{x+3}{4}$

**QUESTION 2**

**EXPAND BINOMIAL PRODUCTS USING A VARIETY OF STRATEGIES**

Expand and simplify the following:

a) $(y + 7)^2$

b) $(3y – 4)^2$

c) $(2x – 1)(2x + 1)$

d) $(3y + 2)(4 – y) + 4y + 5$

e) $(2x – y)^2 – (2x + y)^2$

**QUESTION 3** **FACTORISE ALGEBRAIC EXPRESSIONS**

Factorise the following:

a) $4y^2 + 12y$

b) $y(x + 3) + 8(x + 3)$

c) $2xy + 8x + 4y + 16$

d) $a^2 - 16$

e) $4y^2 - 25x^2$

f) $3d^3 - 3d$

g) $y^2 + 6y + 9$

h) $x^2 + 9x - 36$

i) $6y^2 + 20y + 14$

j) $4x^2 - 20x + 25$

**QUESTION 4** **SIMPLIFY ALGEBRAIC FRACTIONS**

Simplify:

a) $\frac{x^2 + 3x + 2}{x + 2}$

b) $\frac{3m - 6}{4} \times \frac{8m}{m^2 - 2m}$

c) $\frac{4}{x^2 - 9} \div \frac{2}{3x + 9}$

d) $\frac{1}{y - 2} - \frac{2}{y}$

e) $\frac{4}{x^2 + x} - \frac{3}{x^2 - 1}$

# Mathematics Stage 5.3: Indices

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### NUMBER AND ALGEBRA: INDICES

| STUDENTS HAVE LEARNT TO | SYLLAVBUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Convert algebraic expressions with negative indices to expressions with positive indices and vice versa | |
| ☐ Simplify algebraic expressions involving positive, negative and zero indices | |

## STAGE 5.3: INDICES

### SYLLABUS REFERENCE

**MA5.3-6NA** - A student performs operations with surds and indices.

### QUESTION 1

Define a rational number:

_____

_____

_____

## QUESTION 2

**DISTINGUISH BETWEEN RATIONAL AND IRRATIONAL NUMBERS**

Determine whether the following numbers are rational or irrational:

a) 0.32 _____

b) $2\frac{2}{3}$ _____

c) $\frac{\pi}{2}$ _____

d) $\sqrt[3]{34}$ _____

e) $0.\dot{7}$ _____

## QUESTION 3

**WRITE RECURRING DECIMALS IN FRACTION FORM**

Write the following recurring decimals as fractions:

a) $0.\dot{2}$

b) $0.\dot{4}\dot{3}$

c) $1.3\dot{5}\dot{2}$

## QUESTION 4

PERFORM OPERATIONS WITH SURDS

Simplify the following surds:

a) $(\sqrt{8})^2$ _____

b) $(-4\sqrt{6})^2$ _____

c) $\sqrt{\frac{81}{64}}$ _____

d) $\sqrt{2} \times \sqrt{3}$ _____

e) $\sqrt{8}$ _____

f) $6\sqrt{20}$ _____

g) $2\sqrt{3} + 3\sqrt{12}$ _____

h) $2\sqrt{90} \div \sqrt{15}$ _____

i) $\frac{4\sqrt{12} \times 3\sqrt{6}}{2\sqrt{27}}$ _____

## QUESTION 5

SIMPLIFY EXPRESSIONS INVOLVING SURDS

Simplify the following:

a) $6\sqrt{5} + 2\sqrt{5}$ _____

b) $-5\sqrt{7} + 5\sqrt{2} - 3\sqrt{7}$ _____

c) $4\sqrt{8} + 2\sqrt{32}$ _____

d) $\sqrt{2}(\sqrt{4} + \sqrt{7})$ _____

e) $(2 - 3\sqrt{10})(\sqrt{5} - 4\sqrt{2})$ _____

f) $(3\sqrt{7} - \sqrt{5})^2$ _____

g) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ _____

h) $(2\sqrt{5} + 4)(2\sqrt{5} - 4)$ _____

## QUESTION 6

Explain why: $\sqrt{3} + \sqrt{4} \neq \sqrt{7}$

## QUESTION 7

**RATIONALISE THE DENOMINATORS OF SURDS**

Rationalise the denominator for the following (simplifying answers where necessary):

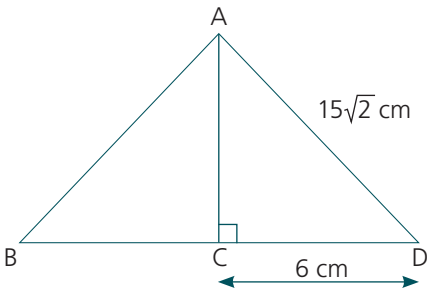
a) $\frac{1}{\sqrt{5}}$

b) $\frac{8}{2\sqrt{10}}$

c) $\frac{2\sqrt{2} - \sqrt{3}}{\sqrt{6}}$

## QUESTION 8

Determine the perpendicular height for the triangle below in exact surd form:



## QUESTION 9

**CONVERT BETWEEN SURD AND INDEX FORM AND VICE VERSA**

Write each of the following with a fractional index:

a) $\sqrt[6]{y}$ _____

b) $\sqrt[3]{64w^9}$ _____

c) $\frac{1}{\sqrt[4]{81x^3}}$ _____

## QUESTION 10

**CONVERT BETWEEN SURD AND INDEX FORM AND VICE VERSA**

Write each of the following in surd form:

a) $x^{\frac{5}{7}}$ _____

b) $4y^{\frac{3}{5}}$ _____

c) $(8m)^{-\frac{3}{2}}$ _____

d) $6y^{-\frac{3}{4}}$ _____

## QUESTION 11

Evaluate the following (answering in exact form):

a) $81^{\frac{3}{4}}$ _____

b) $125^{-\frac{2}{3}}$ _____

c) $16^{-\frac{7}{2}}$ _____

## QUESTION 12

Use a calculator to evaluate the following correct to 3 decimal places:

a) $20^{-\frac{2}{7}}$ _____

b) $9^{\frac{8}{5}}$ _____

c) $\left(\frac{9}{64}\right)^{-\frac{1}{5}}$ _____

# Mathematics Stage 5.3: Equations

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### NUMBER AND ALGEBRA: EQUATIONS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve linear equations involving grouping symbols | |
| ☐ Solve linear equations involving algebraic fractions | |
| ☐ Solve quadratic equations of the form $ax^2 = c$ | |
| ☐ Solve quadratic equations of the form $ax^2 + bx + c = 0$ (where $a = 1$) using factors | |
| ☐ Solve equations resulting from substitution into formulas | |
| ☐ Solve word problems using linear equations | |
| ☐ Solve linear inequalities | |
| ☐ Solve linear simultaneous equations using algebraic and graphical techniques | |

## STAGE 5.3: EQUATIONS

### SYLLABUS REFERENCE

**MA5.3-7NA** - A student solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations.

### QUESTION 1

**SOLVE COMPLEX LINEAR EQUATIONS INVOLVING TWO OR MORE ALGEBRAIC FRACTIONS**

Solve the following:

a) $\frac{3x - 6}{5} - \frac{x + 7}{3} = 2$

b) $\frac{y - 3}{3} - \frac{4y + 3}{2} = \frac{1}{4}$

**QUESTION 2** **SOLVE QUADRATIC EQUATIONS BY FACTORISING**

Solve the following:

a) $x^2 - 16 = 0$

b) $3x^2 = 4$

c) $4x^2 - 8x = 0$

d) $y^2 + 8y + 7 = 0$

e) $x(x - 4) = -4$

f) $(y - 2)^2 = 9$

## QUESTION 3

**SOLVE QUADRATIC EQUATIONS BY COMPLETING THE SQUARE**

Solve the following by completing the square:

a) $p^2 - 6p + 2 = 0$

## QUESTION 4

**SOLVE QUADRATIC EQUATIONS BY USING THE QUADRATIC FORMULA**

Use the quadratic formula to solve the following: (leave answers in surd form)

a) $x^2 - 8x - 4 = 0$

b) $3y^2 - 10y + 4 = 0$

## QUESTION 5

**SUBSTITUTE A PRONUMERAL TO SIMPLIFY HIGHER-ORDER EQUATIONS**

Solve the following by substituting $u = x^2$.

a) $x^4 - 13x^2 + 36 = 0$

b) $4x^4 - 12x^2 - 16 = 0$

## QUESTION 6

**SOLVE SIMPLE CUBIC EQUATIONS OF THE FORM $AX^3 = K$**

Solve the following:

a) $x^3 = -8000$

b) $8x^3 = 216$

c) $11x^3 = 31$

## QUESTION 7

**REARRANGE LITERAL EQUATIONS**

Change the subject of the formulas below to the pronumerals indicated in the brackets:

a) $v = u + at$ (t)

b) $F = \frac{9C}{5} + 32$ (C)

c) $x = \sqrt{b^2 - 4ac}$ (b)

## QUESTION 8

**SOLVE SIMULTANEOUS EQUATIONS WHERE ONE EQUATION IS NON-LINEAR**

Solve the following pairs of simultaneous equations:

a) $\begin{bmatrix} y = x^2 - x - 2 \\ y = x + 6 \end{bmatrix}$

b) $\begin{bmatrix} y = x + 2 \\ y = \frac{2}{x} \end{bmatrix}$

# Mathematics Stage 5.3: Linear Relationships

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### NUMBER AND ALGEBRA: LINEAR RELATIONSHIPS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Apply the gradient-intercept form of the equation of a straight line to interpret and graph straight lines | |
| ☐ Apply the properties of the gradients of parallel and perpendicular lines on the Cartesian plane | |

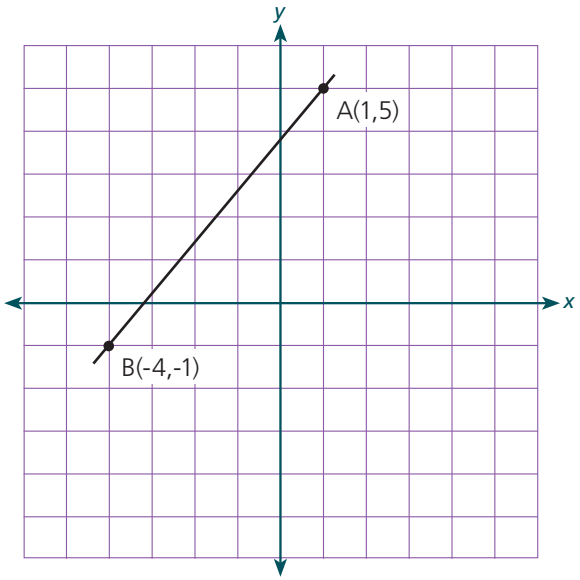
## STAGE 5.3: LINEAR RELATIONSHIPS

### SYLLABUS REFERENCE

**MA5.3-8NA** - A student uses formulas to find midpoint, gradient and distance on the Cartesian plane, and applies standard forms of the equation of a straight line.

### QUESTION 1

**USE FORMULAS TO FIND THE MIDPOINT, GRADIENT AND LENGTH OF INTERVALS ON THE CARTESIAN PLANE**



## QUESTION 2

**USE FORMULAS TO FIND THE MIDPOINT, GRADIENT AND LENGTH OF INTERVALS ON THE CARTESIAN PLANE**

For the interval AB find the following:

a) the midpoint.

b) the gradient.

c) the length.

## QUESTION 3

**APPLY VARIOUS STANDARD FORMS OF THE EQUATION OF A STRAIGHT LINE**

a) Find the equation of the line that has a gradient of -2 and passes through the point (2,3). Leave answer in gradient intercept form.

b) Find the equation of the line that passes through the points A(-3,-1) and B(2,5). Leave answer in general form.

## QUESTION 4

**SOLVE PROBLEMS INVOLVING PARALLEL AND PERPENDICULAR LINES**

a) Find the equation of the line that is parallel to the line $y = 2x + 1$ and passes through the point (3,2).

b) Determine whether the line $y = 3x + 1$ is perpendicular to the line $x + 3y - 6 = 0$.

## QUESTION 5

**SOLVE A VARIETY OF PROBLEMS BY APPLYING COORDINATE GEOMETRY FORMULAS**

a) Show that the points A (-1,4), B(3,1) and C(2,8) form a right-angled triangle.

b) Show that the points P(-7,2), Q(2,-7), R(5,-4) and S(-4,5) form a rectangle.

# Mathematics Stage 5.3: Non-Linear Relationships

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### NUMBER AND ALGEBRA: NON-LINEAR RELATIONSHIPS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Identify, draw and compare graphs of parabolas of the form $y = ax^2 + c$ | |
| ☐ Identify, graph and compare exponential curves and circles | |

## STAGE 5.3: NON-LINEAR RELATIONSHIPS

### SYLLABUS REFERENCE

**MA5.3-9NA** - A student sketches and interprets a variety of non-linear relationships.

### QUESTION 1

For the parabola $y = x^2 - 2x - 3$:

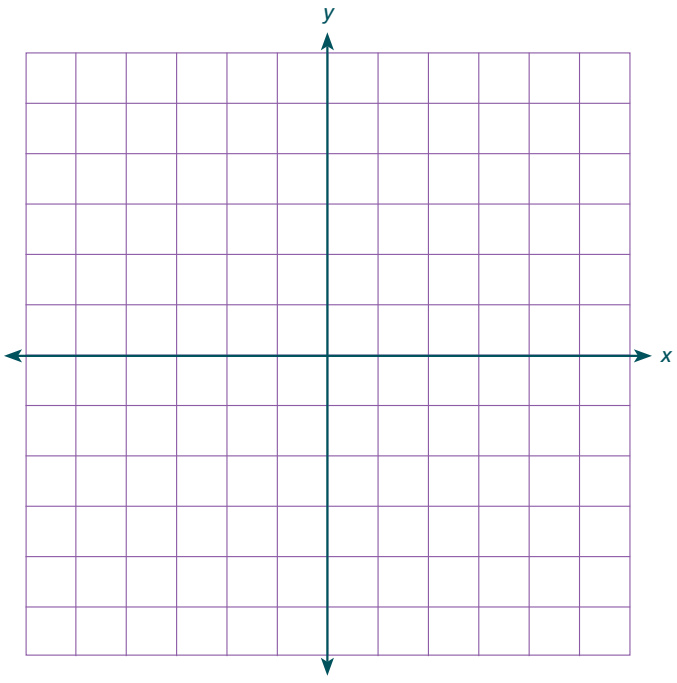
a) find the x and y intercepts.

b) find the equation of the axis of symmetry.

c) determine the minimum value.

d) state the coordinates of the vertex.

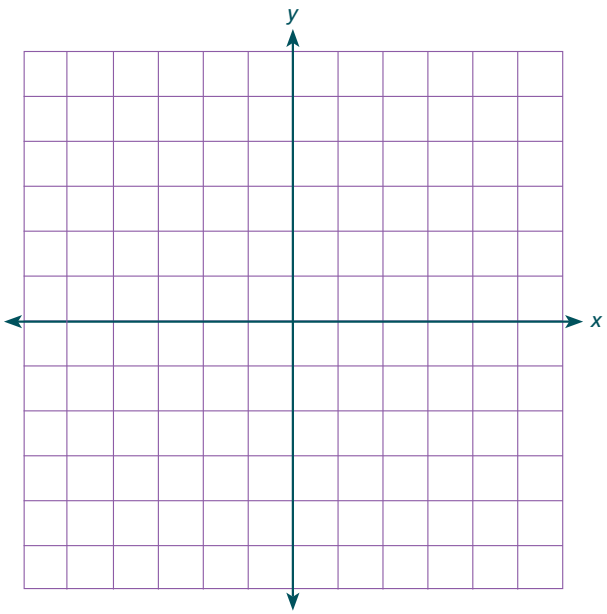
e) Graph the parabola on a Cartesian plane.



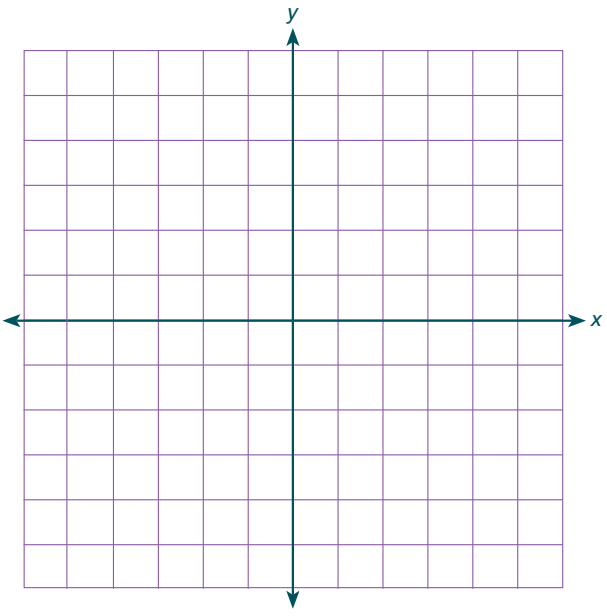
## QUESTION 2

Sketch and describe the similarities and differences of the graphs and what causes those differences:

a) $y = \frac{1}{x}$ $y = \frac{-1}{x}$ $y = \frac{3}{x}$




b) $y = \frac{1}{x}$ $y = \frac{1}{x} + 1$ $y = \frac{1}{x-2}$



## QUESTION 3

State the asymptotes of the following hyperbolas:

a) $y = \frac{1}{x} + 4$ _____

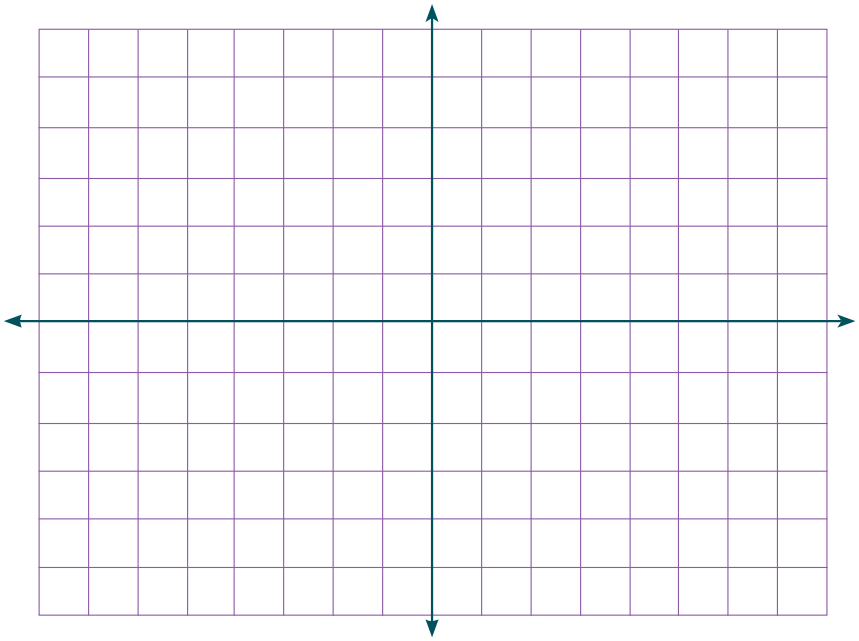
b) $y = \frac{1}{x+3}$ _____

## QUESTION 4

**DESCRIBE, INTERPRET AND SKETCH CUBIC FUNCTIONS**

Graph and compare the features of the following functions:

$y = x^3$ $y = 2x^3$ $y = x^3 + 2$ $y = (x - 4)(x - 2)(x + 1)$



## QUESTION 5

**RECOGNISE AND DESCRIBE EQUATIONS THAT REPRESENT CIRCLES WITH CENTRE (H,K) AND RADIUS R**

a) Determine the centre and radius of the circle $(x - 2)^2 + (y + 3)^2 = 16$.

b) Find the centre and radius of the circle: $x^2 + 2x + y^2 - 2y + 1 = 0$ by completing the square.

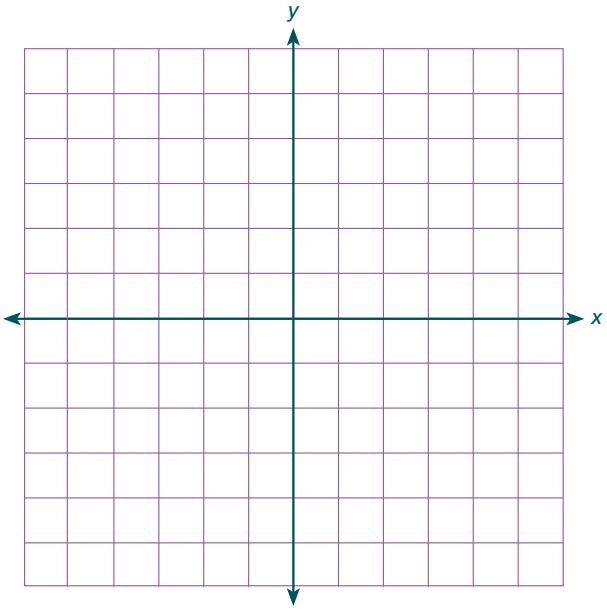
## QUESTION 6

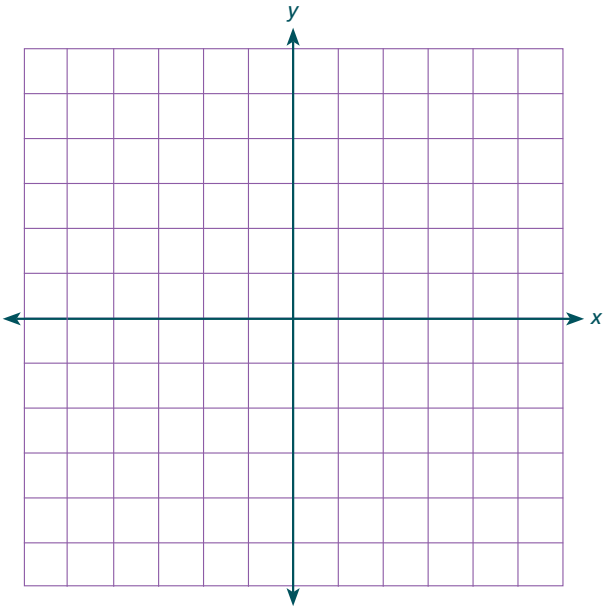
**GRAPH CURVES OF THE FORM $Y = AX^N + K$**

a) Graph the following on a separate number plane by first determining the x and y intercepts.

i) $y = x^5 + 3$

ii) $y = x^4 + 1$



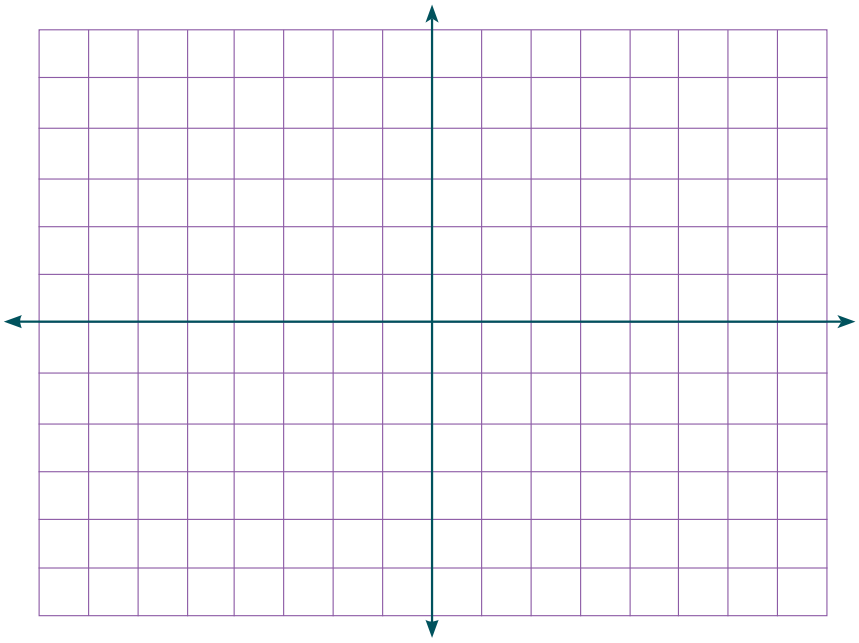




b) What do you notice about the odd and even powers of these functions and their graphs?

## QUESTION 7

**DETERMINE THE POINTS OF INTERSECTION OF A LINE WITH A PARABOLA**

Determine the point of intersection of the parabola $y = x^2 + 1$ and the line $y = -2x + 4$ both graphically and algebraically.

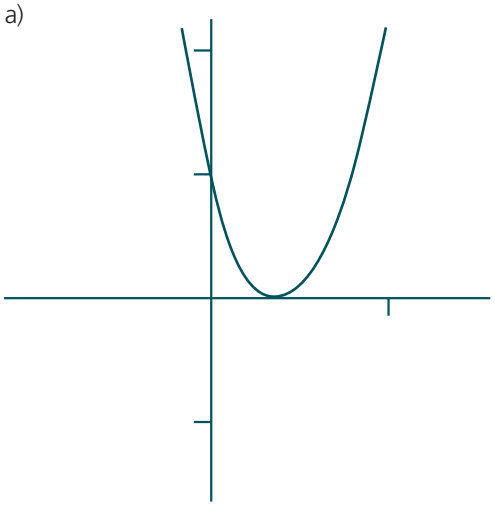


## QUESTION 8

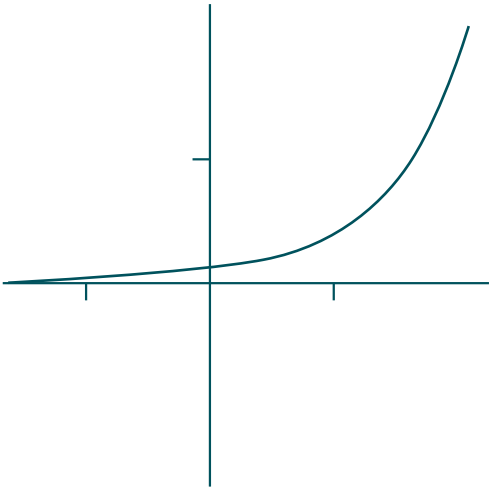
DETERMINE THE POINTS OF INTERSECTION OF A LINE WITH A PARABOLA

Determine a possible equation for each given graph:

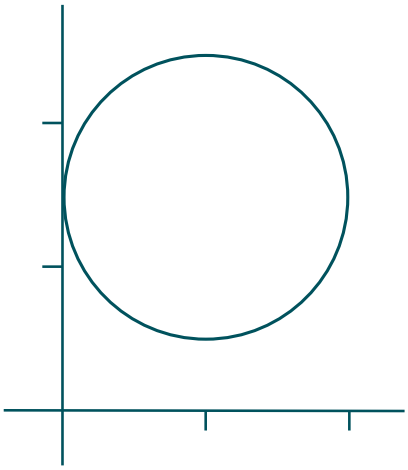
a)



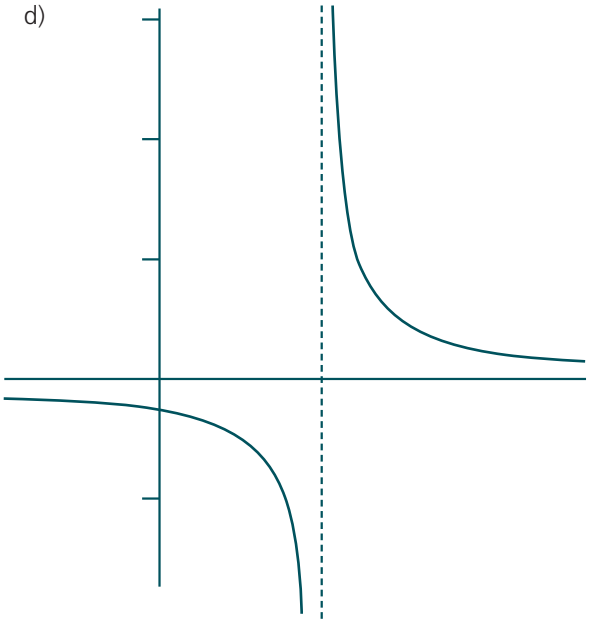
b)



c)



d)



# Mathematics Stage 5.3: Polynomials

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.3 KEY IDEAS

| NUMBER AND ALGEBRA: POLYNOMIALS<br>STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Draw, interpret and compare graphs of parabolas of the form $y = ax^2 + bx + c$ using a variety of techniques | |
| ☐ Determine the equation of the axis of symmetry and the coordinates of the vertex of a parabola | |
| ☐ Draw, interpret and compare graphs of hyperbolas, exponentials, circles and simple cubic functions | |

## STAGE 5.3: POLYNOMIALS

### SYLLABUS REFERENCE

**MA5.3-10NA** - A student recognizes, describes and sketches polynomials, and applies the factor and remainder theorems to solve problems.

### QUESTION 1

**RECOGNISE A POLYNOMIAL EXPRESSION AND USE THE TERMS 'DEGREE', 'LEADING TERM', 'LEADING COEFFICIENT', 'CONSTANT TERM' AND 'MONIC POLYNOMIAL' APPROPRIATELY**

Determine the following for the polynomial $y = 2x^5 - 3x^3 + 5x + 9$:

a) the leading term. _____

b) the degree. _____

c) the leading coefficient. _____

d) is it monic. _____

e) the constant term. _____

**QUESTION 2** **ADD, SUBTRACT, MULTIPLY AND DIVIDE POLYNOMIALS**

Evaluate the following for the polynomials $P(x) = 3x^3 + 2x^2 + 5$ and $Q(x) = x^3 - 2$:

a) $P(x) + Q(x)$

b) $P(x) - Q(x)$

**QUESTION 3** **ADD, SUBTRACT, MULTIPLY AND DIVIDE POLYNOMIALS**

Evaluate the following:

$(2x - 4) \times (5x^2y + 4xy - x)$

**QUESTION 4** **DIVIDE POLYNOMIALS BY LINEAR EXPRESSIONS**

Divide $P(x) = x^3 - 2x + 4$ by $A(x) = (x - 2)$. Then express $P(x)$ in the form $P(x) = A(x) \cdot Q(x) + R(x)$.

**QUESTION 5** **APPLY THE FACTOR THEOREM**

Find the remainder when:

a) $(2y^2 + 7y - 2) \div (y + 7)$

b) $(m^5 + 5m^3 - 1) \div (m - 3)$

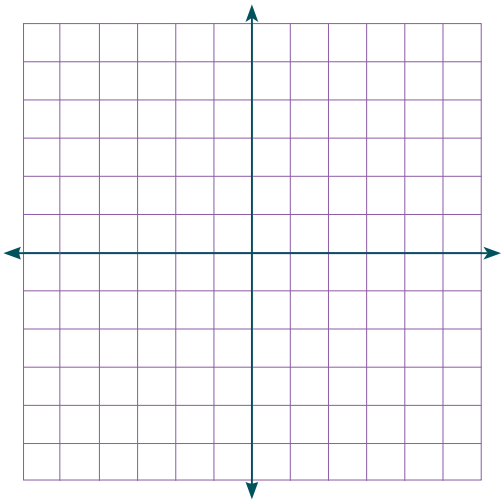
**QUESTION 6** **APPLY THE REMAINDER THEOREM**

Show that $p - 2$ is a factor of $p^3 - 4p^2 + p + 6$.

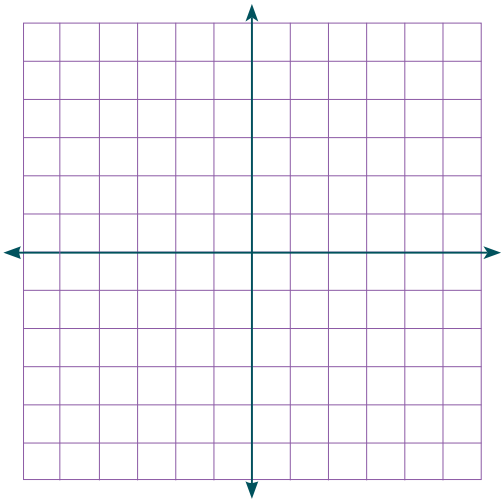
## QUESTION 7

SKETCH A RANGE OF POLYNOMIALS

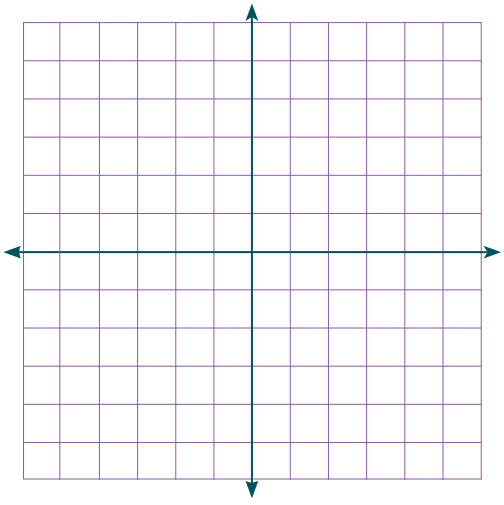
Find the zeros of the following polynomials and sketch the graph of each:



a) $y = (x + 2)(x - 3)(x - 4)$



b) $y = x^3 + 2x^2 + x$



c) $y = 2x^3 + 3x^2 - 12x - 13$

# Mathematics Stage 5.3: Logarithms

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.3 KEY IDEAS

### NUMBER AND ALGEBRA: LOGARITHMS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Draw, interpret and compare graphs of parabolas of the form $y = ax^2 + bx + c$ using a variety of techniques | |
| ☐ Determine the equation of the axis of symmetry and the coordinates of the vertex of a parabola | |
| ☐ Draw, interpret and compare graphs of hyperbolas, exponentials, circles and simple cubic functions | |

## STAGE 5.3: LOGARITHMS

### SYLLABUS REFERENCE

**MA5.3-11NA** - A student uses the definition of a logarithm to establish and apply the laws of logarithms.

### QUESTION 1

**EXPRESS INDEX STATEMENTS IN LOGARITHMIC FORM**

Translate the following into logarithmic expressions:

a) $16 = 4^2$ _____

b) $\frac{1}{3} = 3^{-1}$ _____

c) $4^{\frac{3}{2}} = 8$ _____

## QUESTION 2

**SIMPLIFY EXPRESSIONS USING THE LAWS OF LOGARITHMS**

Apply the law of logarithms to simplify the following expressions:

a) $\log_3 27$

b) $\log_{25} 5$

c) $\log_4 8 + \log_4 2$

d) $\log_{20} 50 + 3\log_{20} 2$

e) $\log_4 16 - 2\log_4 2$

f) $4\log_a a - \log_a a^3$

## QUESTION 3

**SOLVE SIMPLE EQUATIONS THAT INVOLVE EXPONENTS OR LOGARITHMS**

Solve the following:

a) $4^t = 64$

b) $3^{t+1} = 9$

c) $\log_{10} 1000 = x$

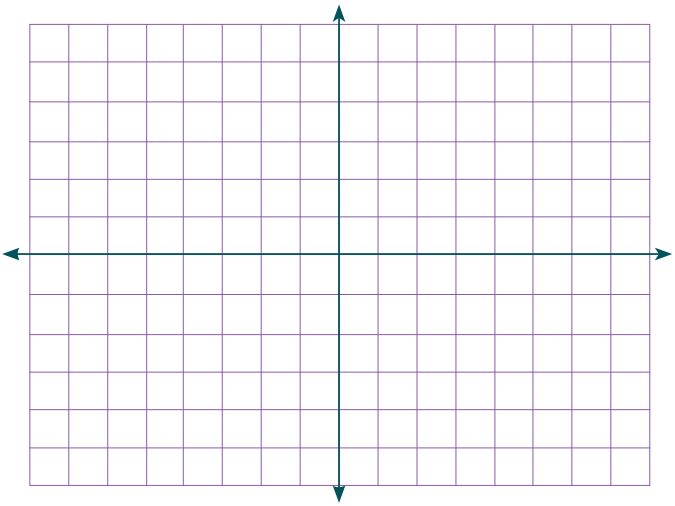
d) $\log_3 x = 5$

## QUESTION 4

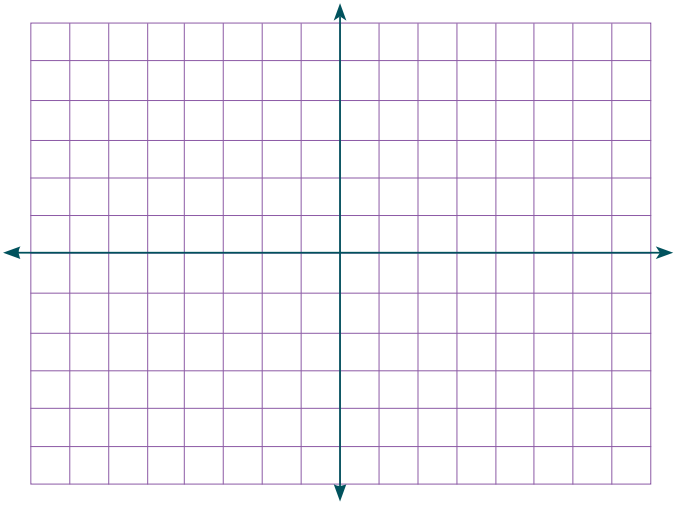
**GRAPH AND COMPARE THE GRAPHS OF THE INVERSE FUNCTIONS $Y = A^x$ AND $Y = \log_A X$**

Graph and compare the features of the following functions:

a) $y = 2^x$ and $y = \log_2 x$



b) $y = 3^x$ and $y = \log_3 x$



# Mathematics Stage 5.3: Functions and Other Graphs

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.3 KEY IDEAS

### NUMBER AND ALGEBRA: FUNCTIONS AND OTHER GRAPHS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Draw, interpret and compare graphs of parabolas of the form $y = ax^2 + bx + c$ using a variety of techniques | |
| ☐ Determine the equation of the axis of symmetry and the coordinates of the vertex of a parabola | |
| ☐ Draw, interpret and compare graphs of hyperbolas, exponentials, circles and simple cubic functions | |

## STAGE 5.3: FUNCTIONS AND OTHER GRAPHS

### SYLLABUS REFERENCE

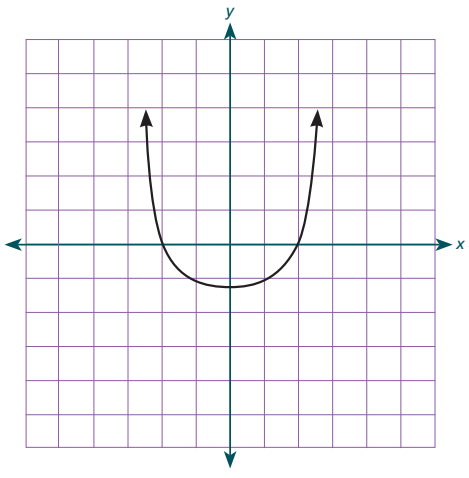
**MA5.3-12NA** - A student uses function notation to describe and sketch functions.

### QUESTION 1

DECIDE WHETHER A GRAPH REPRESENTS A FUNCTION

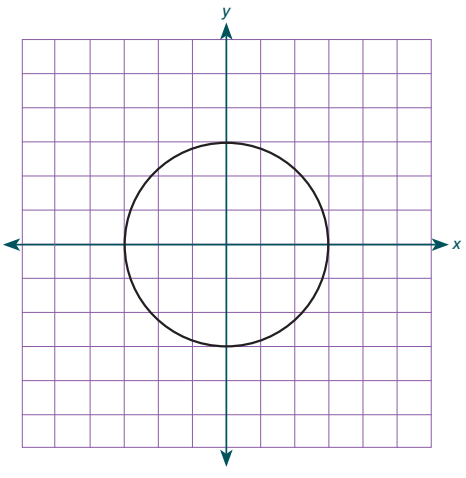
Determine whether the following graphs are functions:

a)



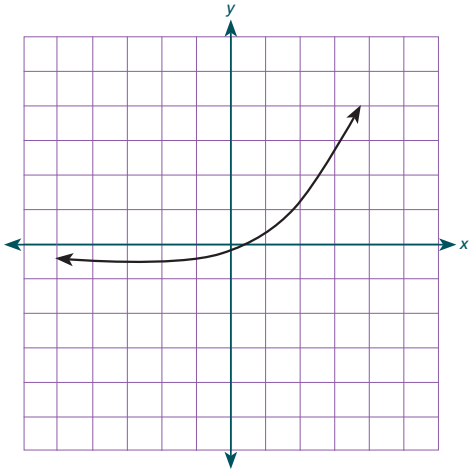

_____

b)

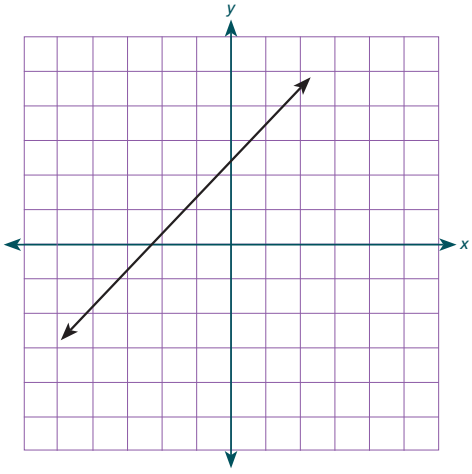



_____

c)




d)




**QUESTION 2** **USE FUNCTION NOTATION**

a) If $f(x) = y^2 + 2y + 1$, find f(-2) and f(1).

b) If $f(x) = 2x + 3$, find x when f(x) = 15.

## QUESTION 3

**DETERMINE THE INVERSE FUNCTIONS FOR A VARIETY OF FUNCTIONS**

Determine whether the following have inverse functions; if so find its inverse:

a) $f(x) = 2x - 3$

b) $f(x) = -\frac{1}{3}x + 2$

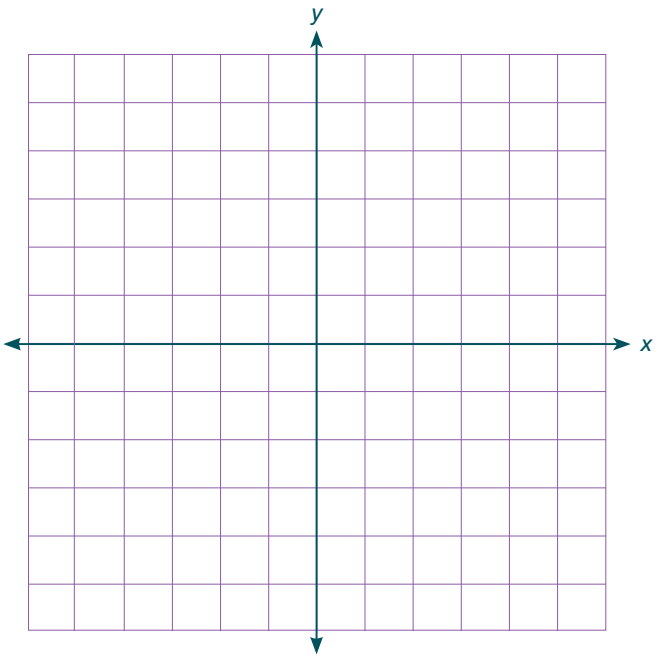
c) $f(x) = 3x^2 - 4$

d) $f(x) = \frac{-x - 2}{x + 4}$

## QUESTION 4

**DETERMINE THE INVERSE FUNCTIONS FOR A VARIETY OF FUNCTIONS AND RECOGNISE THEIR GRAPHS**

Find the inverse $f^{-1}(x)$ of the function $f(x) = x + 1$ and hence sketch both functions on the same Cartesian plane. What do you notice?

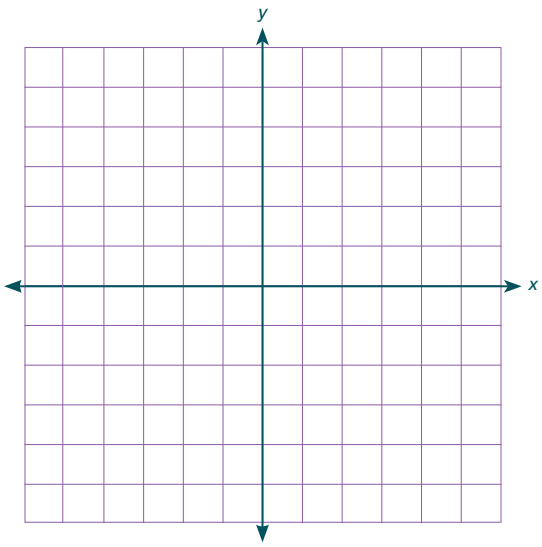


## QUESTION 5

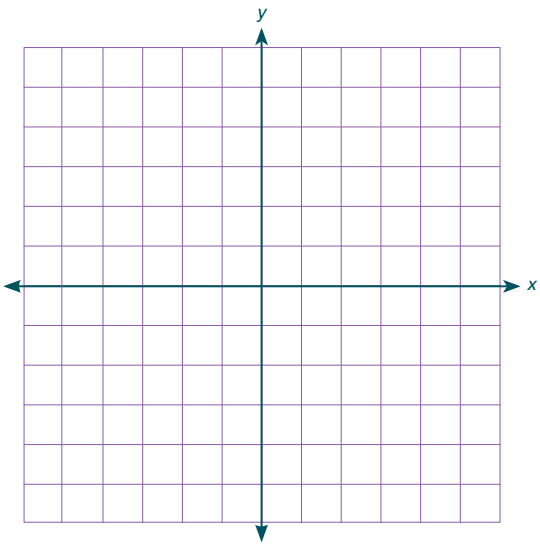
**SKETCH THE GRAPHS OF Y = F(X) + K AND Y = F(X – A), GIVEN THE GRAPH OF Y = F(X)**

If $f(x) = x^2$, then graph:

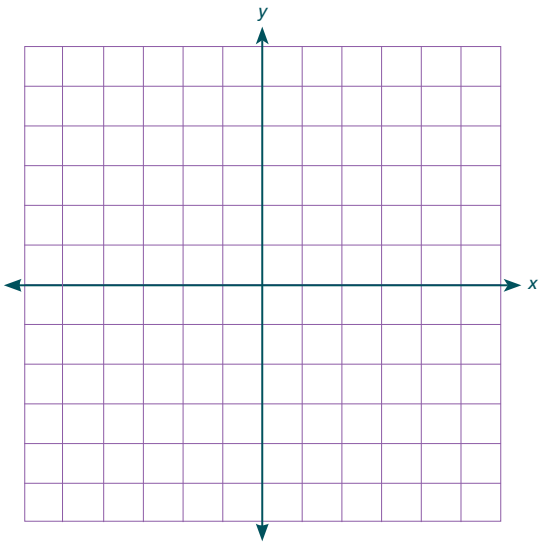
a) $y = f(x)$




b) $y = f(x) + 2$




c) $y = f(x – 4)$



# Mathematics Stage 5.3: Area and Surface Area

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### MEASUREMENT AND GEOMETRY: AREA AND SURFACE AREA

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve problems involving surface area of right prisms, cylinders and related composite solids | |

## STAGE 5.3: AREA AND SURFACE AREA

### SYLLABUS REFERENCE

**MA5.3-13MG** - A student applies formulas to find the surface areas of right pyramids, right cones, spheres and related composite solids.

### QUESTION 1 — SURFACE AREA OF RIGHT PYRAMIDS

A square pyramid is pictured below:

a) determine the perpendicular height of EF in ΔEAB.

_____

_____

_____

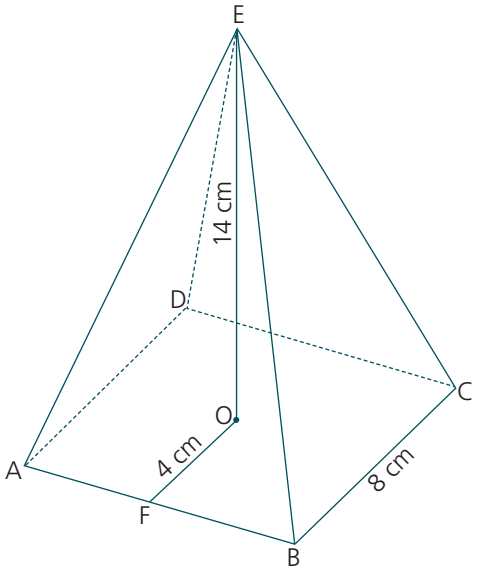
b) calculate the surface area of the pyramid.

_____

_____

_____

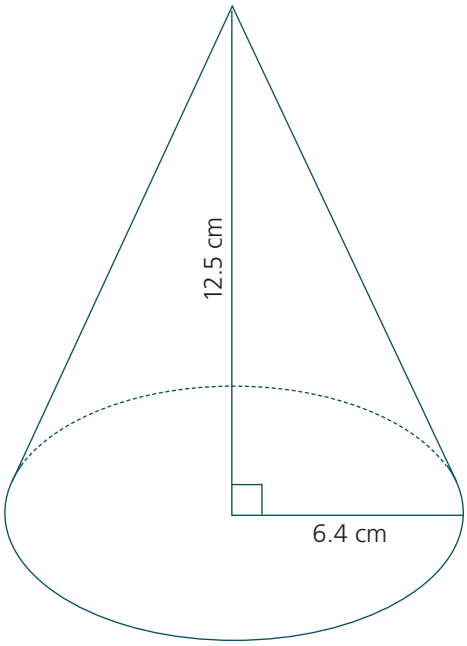
_____



**QUESTION 2** **SURFACE AREA OF A CONE**

a) Calculate the curved surface area of the cone.

b) Find the total surface area of the cone.

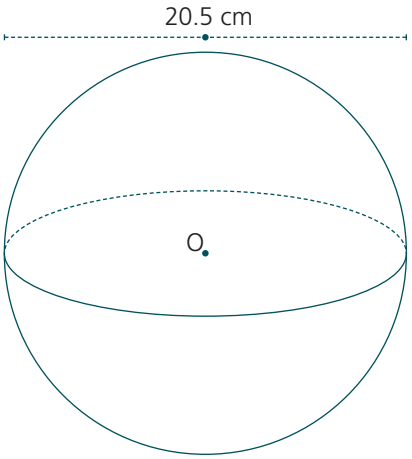


## QUESTION 3

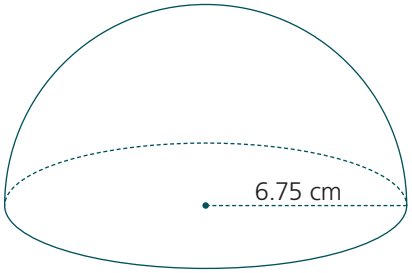
SURFACE AREA OF A SPHERE

a) Calculate the surface area of the following:

i)




ii)



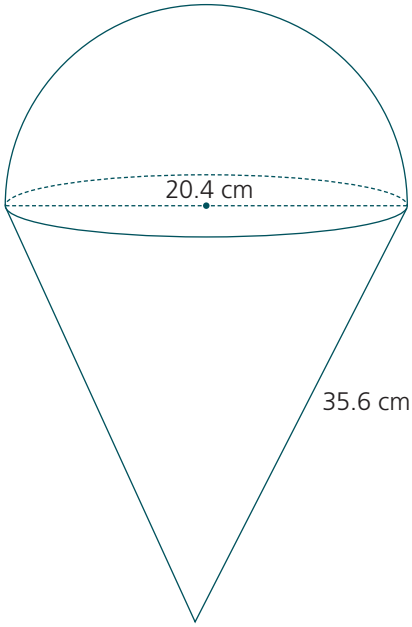

b) The surface area of a sphere is 45 $cm^2$. Calculate the radius of the sphere.

## QUESTION 4

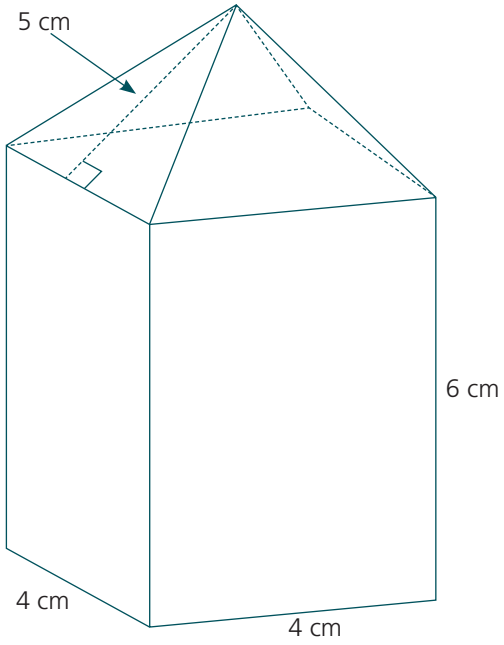
SURFACE AREA OF COMPOSITE SOLIDS

Calculate the surface area of the following composite shapes:

a)



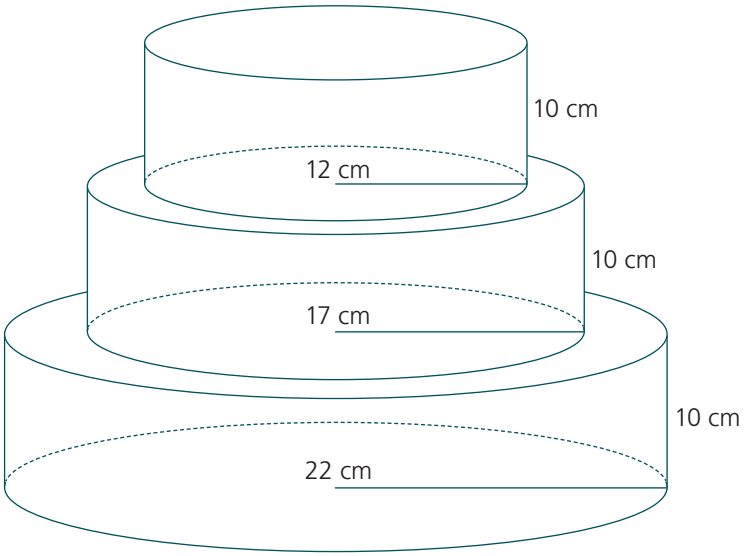

b)



**QUESTION 5**

**SOLVE PRACTICAL PROBLEMS INVOLVING THE SURFACE AREAS OF SOLIDS**

A wedding cake has three tiers as indicated in the diagram below. Each tier is 10 cm high. The layers have radii 22 cm, 17 cm and 12 cm respectively. Find the total surface area of the cake, correct to the nearest $cm^2$.



# Mathematics Stage 5.3: Volume

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### MEASUREMENT AND GEOMETRY: VOLUME

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Solve problems involving volume and capacity of prisms, cylinders and related composite solids | |

## STAGE 5.3: VOLUME

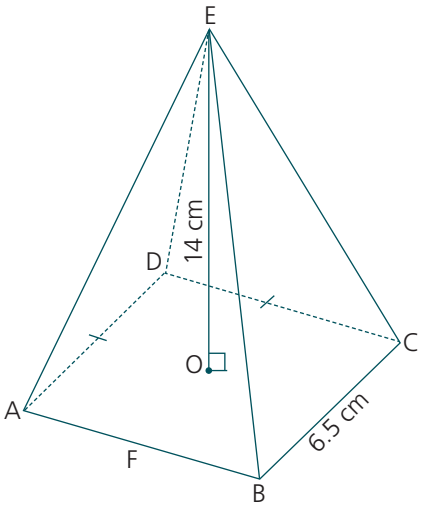
### SYLLABUS REFERENCE

**MA5.3-14MG** - A student applies formulas to find the volumes of right pyramids, right cones, spheres and related composite solids.

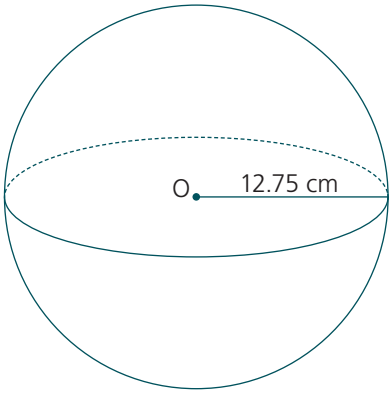
| QUESTION 1 | SOLVE PROBLEMS INVOLVING VOLUME OF RIGHT PYRAMIDS, RIGHT CONES, SPHERES AND RELATED COMPOSITE SOLIDS |
|---|---|

Calculate the volume of the following:

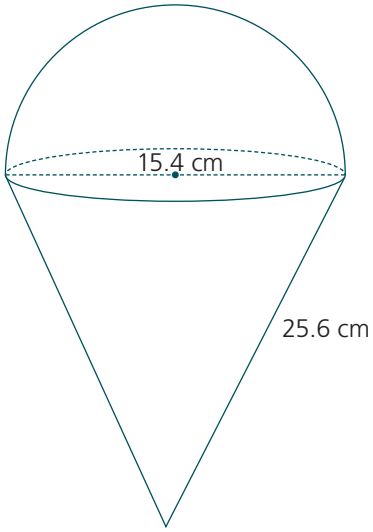
a)



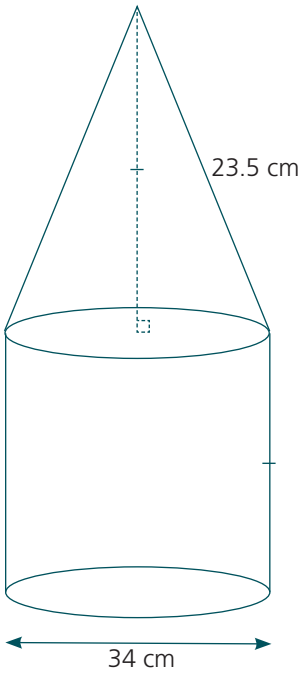
b)




c)




d)



## QUESTION 2

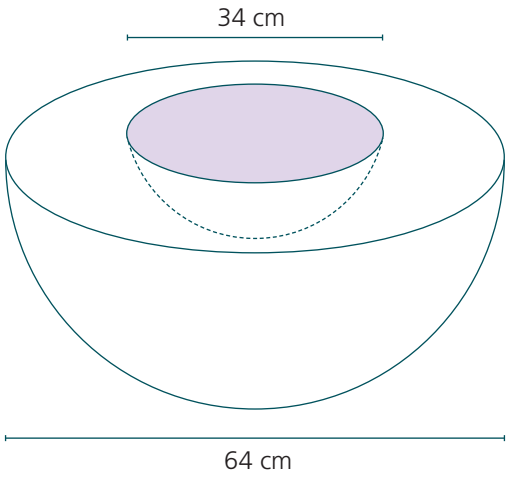
APPLYING VOLUME FORMULAS

Find the radius of the cone whose volume is 1365 $cm^3$ and has a perpendicular height of 35 cm.

## QUESTION 3

VOLUME AND CAPACITY OF COMPOSITE SOLIDS

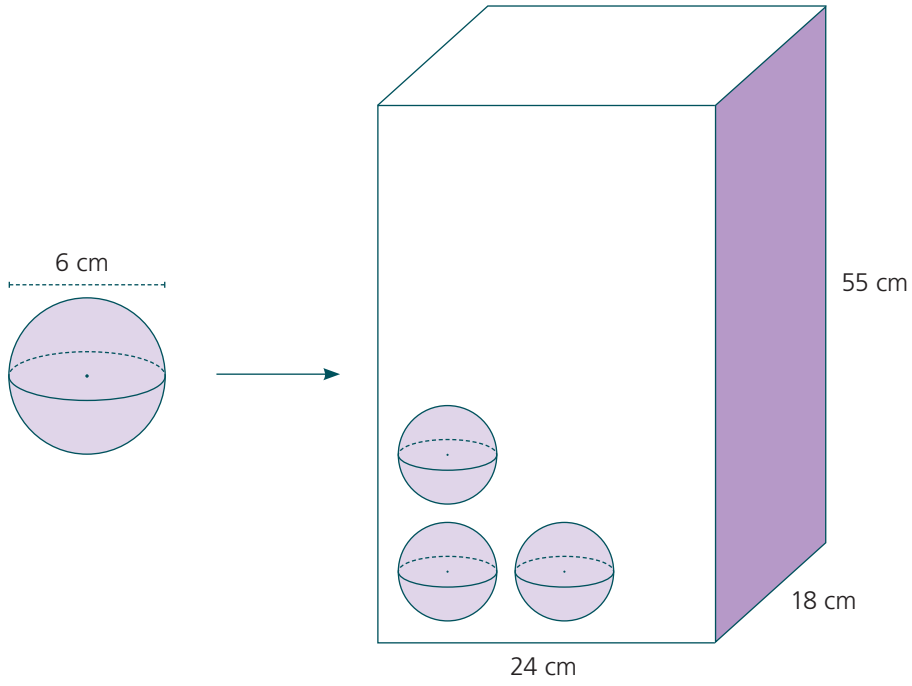
Find the volume of the following and the capacity that it holds to the nearest litre:



**QUESTION 4**

**SOLVE A VARIETY OF PRACTICAL PROBLEMS RELATING TO THE VOLUMES AND CAPACITIES**

A rectangular prism is to fit as many spherical balls as possible:




a) How many spherical balls can it fit?

b) What percentage of the box will be empty?

# Mathematics Stage 5.3: Trigonometry

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### MEASUREMENT AND GEOMETRY: TRIGONOMETRY

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Use trigonometry to find sides and angles in right-angled triangles (including angles measured to nearest minute or second) | |
| ☐ Apply trigonometry to solve word problems involving right-angled triangles, including the use of three-figure bearings and angles of elevation and depression | |

## STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

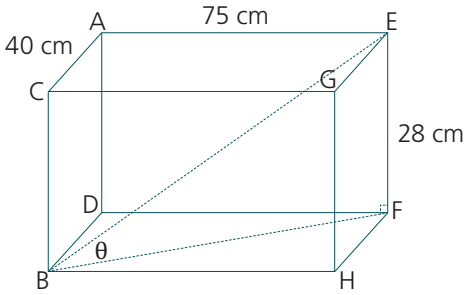
### SYLLABUS REFERENCE

**MA5.3-15MG** - A student applies trigonometry to solve problems, including problems involving bearings.

### QUESTION 1

**SOLVE THREE-DIMENSIONAL PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES**

A rectangular prism has dimensions 28 cm, 40 cm and 75 cm. An umbrella will be packaged in the box below:




a) What is the longest length that the umbrella can be?

_____

_____

b) Calculate the angle θ that BF makes with BE in the plane BDFH.

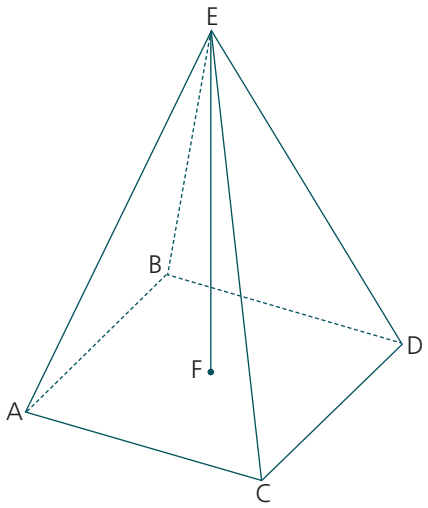
_____

_____

**QUESTION 2**

**SOLVE THREE-DIMENSIONAL PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES**

The pyramid below has a square base ABCD with an area of 16 $cm^2$. ED, EB, EA and EC have a side length of 20 cm. EF is the perpendicular height of the pyramid. Calculate EF in exact form.



**QUESTION 3**

**SOLVE THREE-DIMENSIONAL PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES**

From a point B, due south of the flagpole 150 m tall on level ground, the angle of elevation of the top of the flagpole is 45°. The top of the same flagpole is observed with an angle of elevation of 32° from a point C, due east of the flagpole.

a) Sketch a diagram to portray the above information.

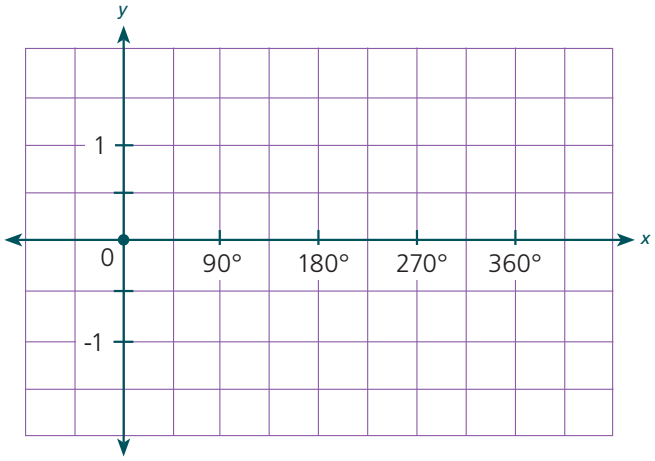
b) Calculate the distance from B to C.

## QUESTION 4

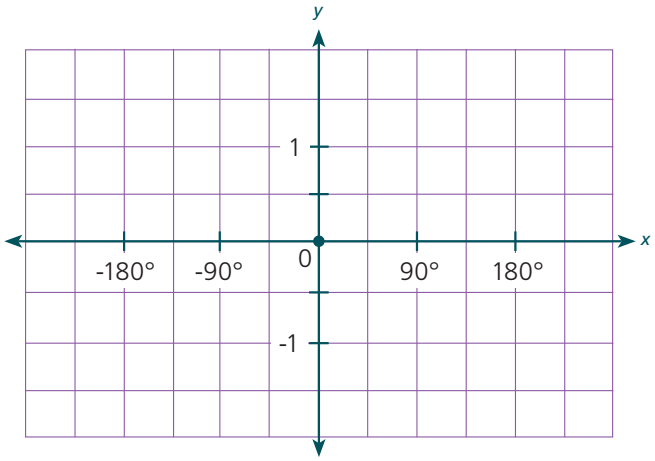
SKETCH TRIGONOMETRIC FUNCTIONS

Sketch the following trigonometric graphs:

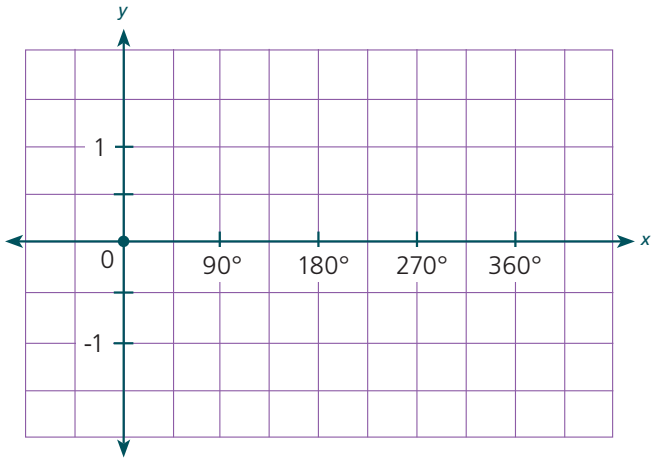
a) $y = \sin x$ for $0° \leq x \leq 360°$




b) $y = \cos x$ for $-180° \leq x \leq 180°$




c) $\tan x$ for $0° \leq x \leq 360°$



## QUESTION 5

USING EXACT TRIGONOMETRIC VALUES

Evaluate the exact values of the following:

a) $\sin 60° + \cos 30°$

b) $\cos^2 60° + \sin^2 30°$

c) $\sin 120°$

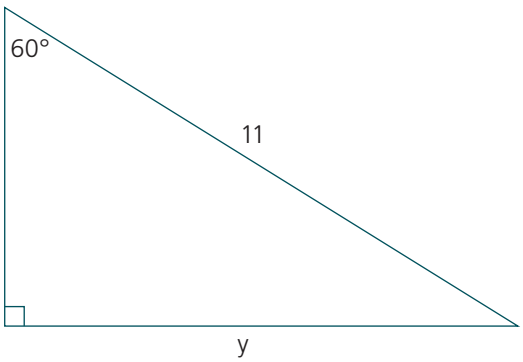
d) $\sin 225°$

e) $\dfrac{\sin 45°}{\cos 60°}$

## QUESTION 6

USING EXACT TRIGONOMETRIC VALUES

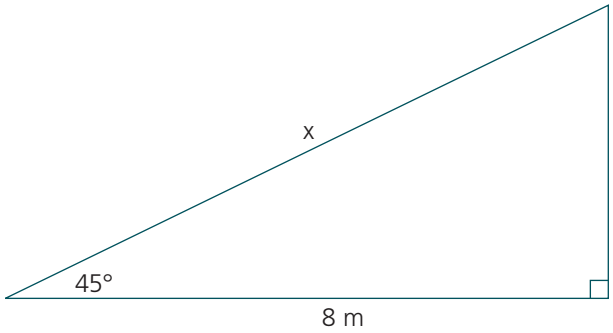
Evaluate the exact length of y.



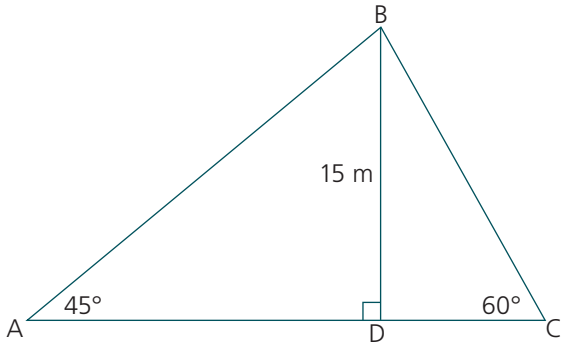
## QUESTION 7

**SOLVE PROBLEMS IN RIGHT-ANGLED TRIANGLES USING EXACT VALUES**

a) A boat ramp is to be made with an angle of 45° and base length 8 m. What is the exact length of the surface of the ramp?




b) Find the exact length of AC.



## QUESTION 8

SOLVE SIMPLE TRIGONOMETRIC EQUATIONS

Solve the following for $0° \leq \theta \leq 360°$:

a) $\sin \theta = \frac{1}{\sqrt{2}}$

b) $2 \cos \theta = \sqrt{3}$

## QUESTION 9

ANGLE OF INCLINATION

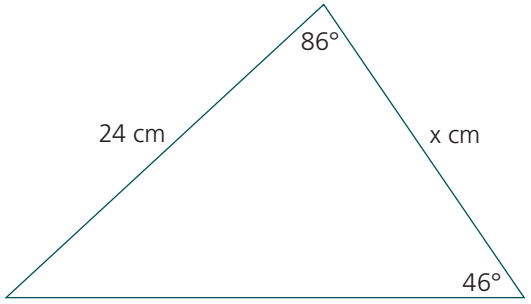
Find the angle of inclination that a line with gradient 0.8 makes with the x-axis in a positive direction to the nearest degree.

## QUESTION 10

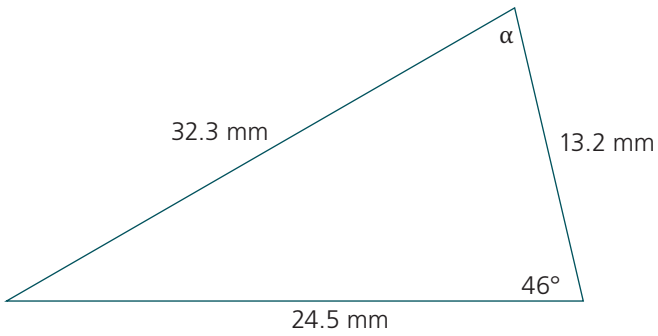
**PROBLEMS INVOLVING NON-RIGHT-ANGLED TRIANGLES**

Find the value of the pronumeral in the following diagrams:

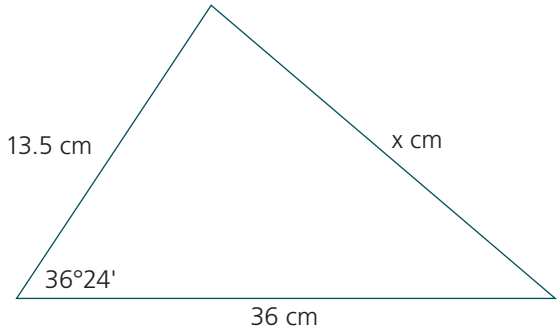
a) (correct to 1 decimal place)



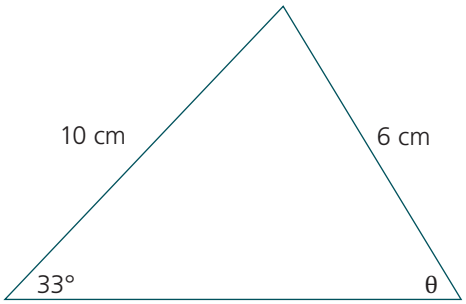

b) (correct to the nearest minute)




c) (correct to 3 significant figure)

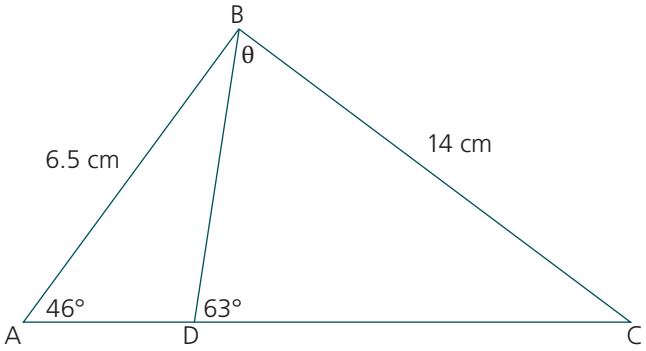


d) (correct to the nearest degree)




**QUESTION 11**

**PROBLEMS INVOLVING MORE THAN ONE TRIANGLE**




a) Find the length of BD.

b) Determine the size of θ to the nearest degree.

**QUESTION 12**

**SOLVE PRACTICAL PROBLEMS THAT INVOLVE NON-RIGHT-ANGLED TRIANGLES**

a) A parallelogram has sides 11 cm and 5 cm, and one interior angle 79°25'. Find the length of the diagonals.

b) A plane flies from Sydney on a bearing of 139° for 852 km, then turns and flies on a bearing of 285° until it is due west of Sydney. How far from Sydney is the plane, to the nearest km?

# Mathematics Stage 5.3: Properties of Geometric Figures

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### MEASUREMENT AND GEOMETRY: PROPERTIES OF GEOMETRIC FIGURES

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Write formal proofs of congruent triangles | |
| ☐ Verify properties of isosceles and equilateral triangles, and special quadrilaterals using congruent triangles | |
| ☐ Determine the minimum conditions for triangles to be similar | |
| ☐ Use deductive reasoning in numerical exercises involving plane shapes | |
| ☐ Establish and use the sum of exterior angles and the sum of interior angles results for polygons | |

## STAGE 5.3: PROPERTIES OF GEOMETRIC FIGURES

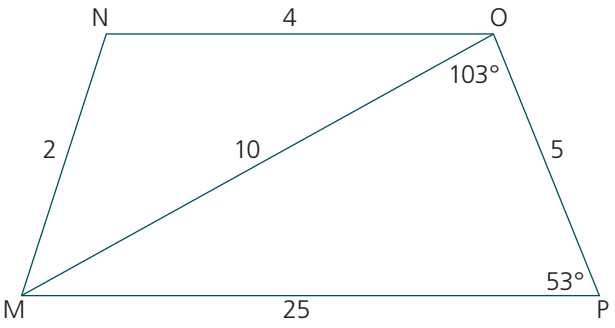
### SYLLABUS REFERENCE

**MA5.3-16MG** - A student calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar.

### QUESTION 1

**WRITE FORMAL PROOFS OF SIMILAR TRIANGLES**

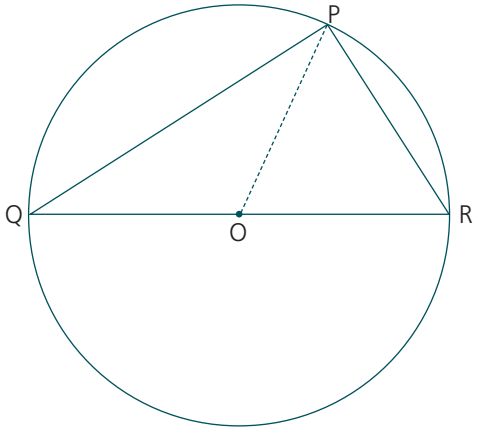
Prove that ΔMNO and ΔPOM are similar.



**QUESTION 2**

**APPLY LOGICAL REASONING TO PROOFS INVOLVING PLANE SHAPES**

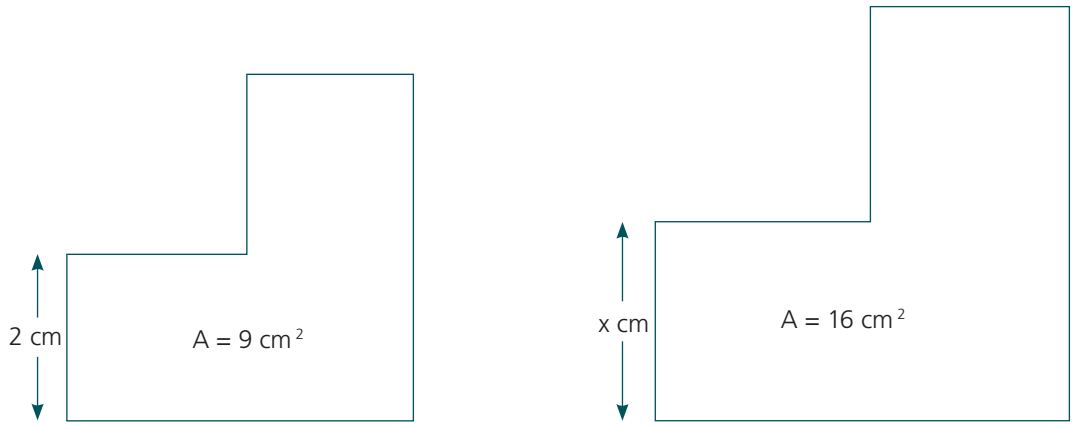
In the diagram below, OQ = OR = OP. Prove that <QPR is right angled.



**QUESTION 3**

**SOLVE PROBLEMS INVOLVING SIMILARITY RATIOS AND AREAS AND VOLUMES**

a) The two composite shapes below are similar, find the value of x.

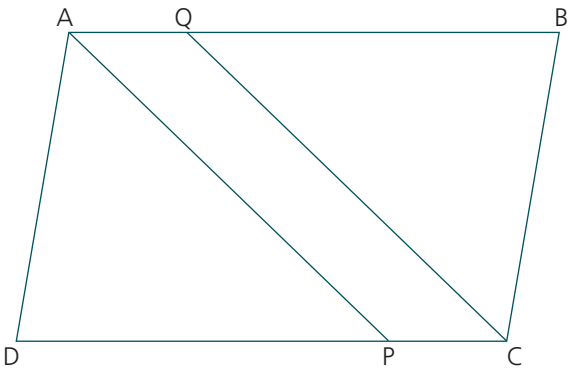



b) Two similar objects have volumes 26 cm $^3$ and 172.75 cm $^3$ respectively. If the smaller object has a length of 2.5 cm, what will be the length of the larger object to 2 significant figures?

## QUESTION 4

**PROVE AND APPLY THEOREMS AND PROPERTIES RELATED TO QUADRILATERALS**

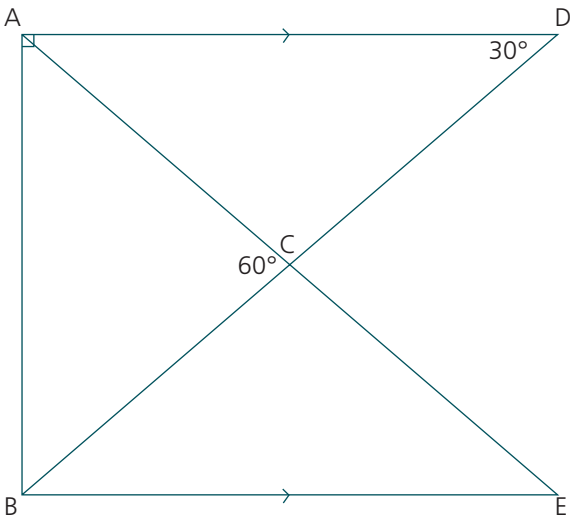
a) ABCD and AQCP are parallelograms. Prove that BQ = DP.




## QUESTION 5

**PROVE AND APPLY THEOREMS AND PROPERTIES RELATED TO TRIANGLES**

Prove that ΔABC is equilateral and ΔBCE is isosceles.



# Mathematics Stage 5.3: Circle Geometry

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### MEASUREMENT AND GEOMETRY: CIRCLE GEOMETRY

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Use the language, notation and conventions of geometry | |
| ☐ Apply the geometric properties of angles at a point to find unknown angles with appropriate reasoning | |
| ☐ Apply the properties of corresponding, alternate and co-interior angles on parallel lines to find unknown angles with appropriate reasoning | |
| ☐ Determine and justify that particular lines are parallel | |
| ☐ Solve simple numerical exercises based on geometrical properties | |

## STAGE 5.3: CIRCLE GEOMETRY

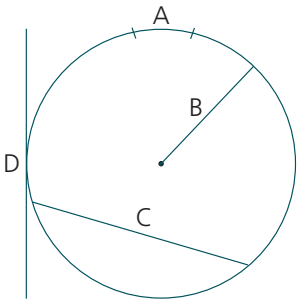
### SYLLABUS REFERENCE

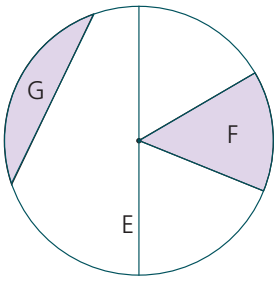
**MA5.3-17MG** - A student applies deductive reasoning to prove circle theorems and to solve related problems.

### QUESTION 1

**IDENTIFY AND NAME PARTS OF A CIRCLE**

Identify the following parts of the circle:




A: _____

B: _____

C: _____

D: _____

E: _____

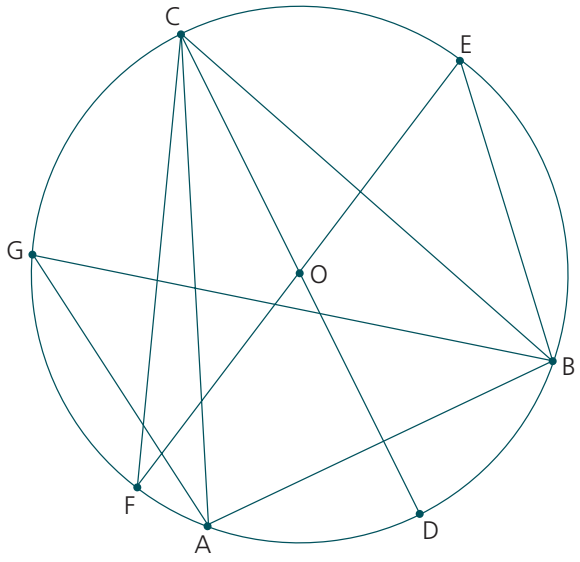
F: _____

G: _____

## QUESTION 2

**USE TERMINOLOGY ASSOCIATED WITH ANGLES IN CIRCLES**

Using the diagram below:




a) identify an angle that subtends from the arc CE. _____

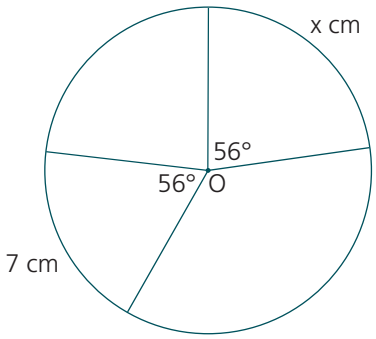
b) identify two angles standing on the arc AB. _____

## QUESTION 3

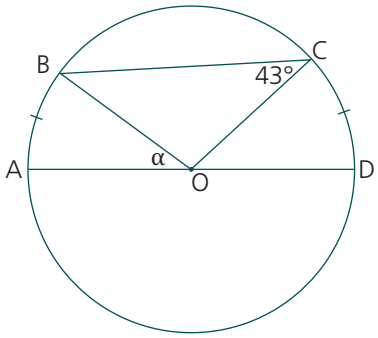
**PROVE AND APPLY ANGLE AND CHORD PROPERTIES OF CIRCLES**

Find the value of the pronumerals in the following and give reasons for your answer:

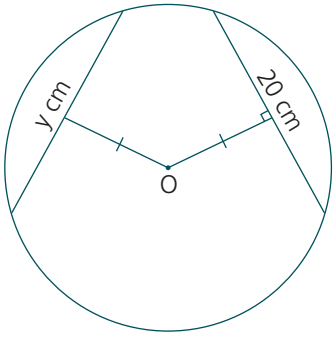
a)



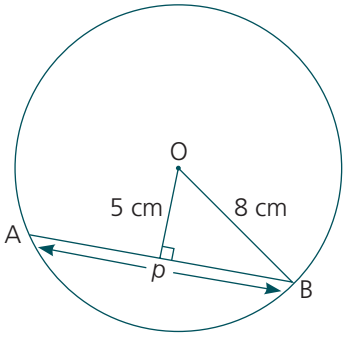

b)



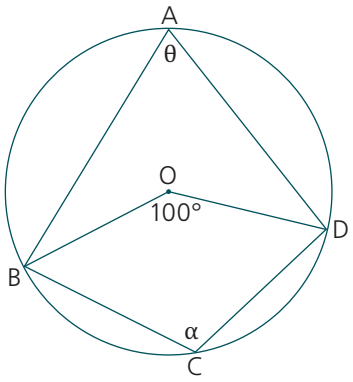

c)



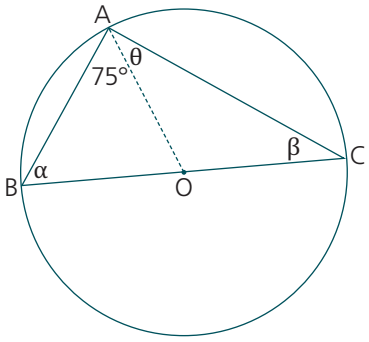

d)



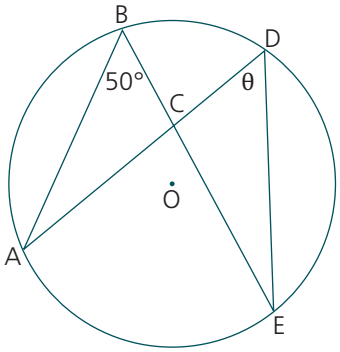
e)



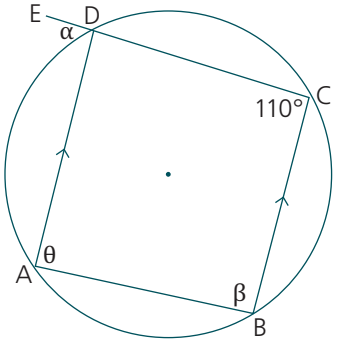

f)




g)



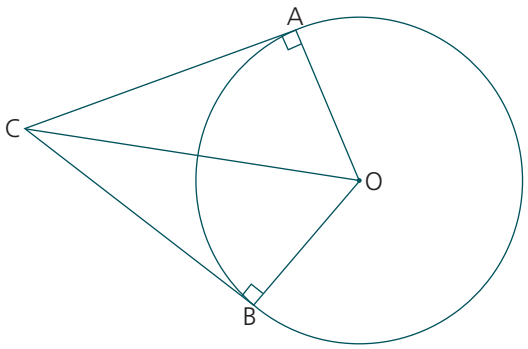

h)



## QUESTION 4

**PROVE AND APPLY TANGENT AND SECANT PROPERTIES OF CIRCLES**

a) In the diagram below:

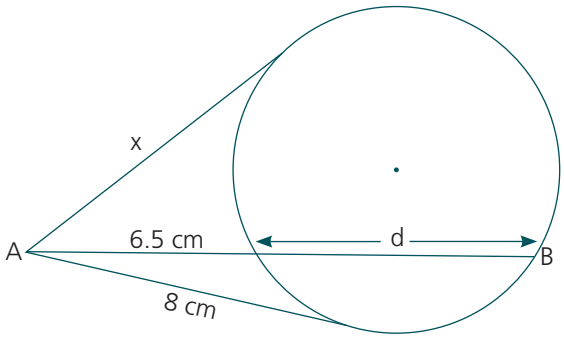



i) prove that $\Delta OAC \equiv \Delta OBC$.

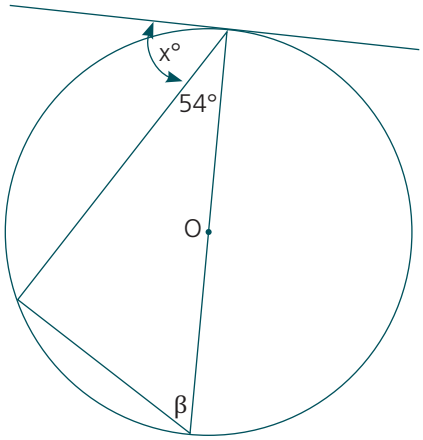
ii) hence, prove that AC = BC.

b) Find the value of the pronumerals in the following diagrams (provide reasons for your answer):

i)




ii)



# Mathematics Stage 5.3: Single Variable Data Analysis

Name: _____

Class: _____

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### STATISTICS AND PROBABILITY: SINGLE VARIABLE DATA ANALYSIS

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Determine quartiles and interquartile range | |
| ☐ Construct, interpret and use box plots to compare sets of data | |
| ☐ Compare shapes of box plots to corresponding histograms and dot plots | |
| ☐ Critically evaluate sources of data in media reports and elsewhere | |

## STAGE 5.3: SINGLE VARIABLE DATA ANALYSIS

### SYLLABUS REFERENCE

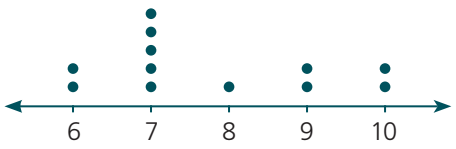
**MA5.2-18SP** - A student uses standard deviation to analyse data.

### QUESTION 1 — CALCULATING STANDARD DEVIATION

Calculate the standard deviation for the following set of data:

a) { 3  5  5  7  9  10  20 } _____

b)




_____

## QUESTION 2

**COMPARING DATA SETS**

The following back to back stem-and-leaf plot shows the marks for English and Maths for a class out of 50.

| MATHS | STEM | ENGLISH |
|---|---|---|
| | 0 | 6 |
| 9 | 1 | 1 4 |
| 8 2 | 2 | 2 5 |
| 8 7 5 | 3 | 7 7 |
| 9 6 6 | 4 | 1 |
| | 5 | 0 |

a) Complete the table for each data set.

| SUBJECT | MEAN | STANDARD DEVIATION |
|---|---|---|
| ENGLISH | | |
| MATHS | | |

b) Which subject did the class perform better in?
Comment on the spread and mean of each subject.

# Mathematics Stage 5.3: Bivariate Data Analysis

Name: _____

Class: _____

**BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS**

**STATISTICS AND PROBABILITY: BIVARIATE DATA ANALYSIS**

| STUDENTS HAVE LEARNT TO | SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME |
|---|---|
| ☐ Construct and interpret displays of bivariate numerical data where the independent variable is time | |
| ☐ Construct and interpret scatter plots of two numerical variables | |

## SYLLABUS REFERENCE

**MA5.2-19SP** - A student investigates relationships between numerical variables using line of best fit, and explores how data is used to inform decision-making processes.

## QUESTION 1

**INVESTIGATE BIVARIATE NUMERICAL DATA SETS**

The local store kept a record of how much slushies they sell versus the average temperature for the day. The results are displayed below:

| TEMPERATURE (T) | 15° | 17° | 16° | 25° | 27° | 22° | 28° | 30° | 35° | 12° | 26° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SLUSHIES SOLD (S) | 12 | 25 | 47 | 58 | 130 | 60 | 80 | 140 | 70 | 10 | 100 |

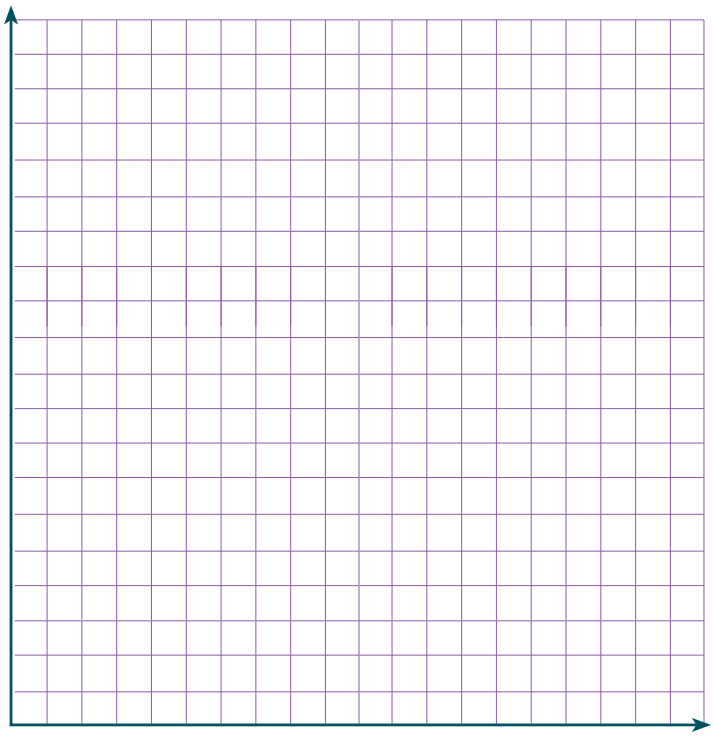
a) enter the data into a spreadsheet and create a scatterplot (use the graph below if you do not have a computer).

b) draw a line of best fit.

c) determine the equation of the line of best fit.

d) determine the value of t, if S = 48. _____

e) determine the value of S, if t = 29. _____

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