MATHEMATICS
STAGE 5
Introduction

Mathematics Stage 5 Diagnostic Tasks have been designed to assist secondary teachers in accurately identifying skills, knowledge and understandings of students in Stage 5. They are intended to be a diagnostic snapshot of learning key ideas and concepts from the NSW Mathematics K-10 Syllabus for the Australian Curriculum. This resource is one example of assessment for learning; other forms of assessment include hands-on tasks, practical activities, investigations, observations and anecdotal evidence. A wide range of assessments are also used by teachers to identify teaching points and learning needs for all students.

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Stage 5.2

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<tr>
<td>15</td>
<td>Single Variable Data Analysis</td>
</tr>
<tr>
<td>16</td>
<td>Bivariate Data Analysis</td>
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</table>
MATHEMATICS
STAGE 5.1
# Mathematics Stage 5.1: Financial Mathematics

Name: 

Class: 

## BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

### NUMBER AND ALGEBRA: COMPUTATION WITH INTEGERS

<table>
<thead>
<tr>
<th>STUDENTS HAVE LEARNT TO</th>
<th>SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Order integers</td>
<td></td>
</tr>
<tr>
<td>□ Add and subtract integers</td>
<td></td>
</tr>
<tr>
<td>□ Multiply and divide integers</td>
<td></td>
</tr>
<tr>
<td>□ Perform order of operations</td>
<td></td>
</tr>
<tr>
<td>□ Use a calculator to perform operations with integers</td>
<td></td>
</tr>
<tr>
<td>□ Solve problems</td>
<td></td>
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### NUMBER AND ALGEBRA: FRACTIONS, DECIMALS AND PERCENTAGES

<table>
<thead>
<tr>
<th>STUDENTS HAVE LEARNT TO</th>
<th>SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Express improper fractions as mixed numerals and vice versa</td>
<td></td>
</tr>
<tr>
<td>□ Generate equivalent fractions, write fractions in simplest form</td>
<td></td>
</tr>
<tr>
<td>□ Apply the four operations with fractions and decimals</td>
<td></td>
</tr>
<tr>
<td>□ Convert between fractions, decimals and percentages</td>
<td></td>
</tr>
<tr>
<td>□ Express one quantity as a fraction/percentage of another</td>
<td></td>
</tr>
<tr>
<td>□ Round decimals to a specified number of places</td>
<td></td>
</tr>
<tr>
<td>□ Use the notation of recurring decimals</td>
<td></td>
</tr>
<tr>
<td>□ Convert fractions into recurring decimals</td>
<td></td>
</tr>
<tr>
<td>□ Solve problems involving fractions, decimals and percentages</td>
<td></td>
</tr>
</tbody>
</table>

### NUMBER AND ALGEBRA: FINANCIAL MATHEMATICS

<table>
<thead>
<tr>
<th>STUDENTS HAVE LEARNT TO</th>
<th>SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Perform calculations involving GST</td>
<td></td>
</tr>
<tr>
<td>□ Calculate discounts and ‘best buys’</td>
<td></td>
</tr>
<tr>
<td>□ Solve problems involving profit and loss</td>
<td></td>
</tr>
</tbody>
</table>
**STAGE 5.1: FINANCIAL MATHEMATICS**

**SYLLABUS REFERENCE**

**MA5.1- 4NA** - A student solves financial problems involving earning, spending and investing money.

---

**QUESTION 1**

**SOLVE PROBLEMS INVOLVING EARNING MONEY**

a) Lucy works a 40 hour week and gets paid $13.50 an hour. What is her total earnings for the week?

b) Julie works 28 hours as a beautician. She earns $365.50 a week. What is her hourly rate of pay?

c) Chris earns $16.30 per hour for a 36 hour week. He receives overtime at time and a half for the first 4 hours and double time thereafter. Calculate Chris's earnings for the week if he works 45 hours.

d) Craig is a builder and earns a salary of $76 500 p.a. Find Craig's:
   
i) fortnightly pay
   
ii) monthly pay

e) Amirita is a computer programmer and earns a salary of $67 500. She decides to take her annual holidays and is paid 4 weeks normal pay plus 17.5% holiday loading. *(Holiday loading is calculated on 4 weeks normal pay)*
   
i) Calculate her holiday loading.
   
ii) Calculate her total holiday pay.
STAGE 5.1: FINANCIAL MATHEMATICS

QUESTION 2

COMMISSION AND PIECE WORK

a) John is a car salesman. He earns a retainer of $350 a week plus 3% commission on the value of car sales exceeding $25 000. John sold a total of $55 000 in car sales this week. What is his total pay for the week?

b) A factory worker earns $1.30 for every wooden toy he makes and 45c for every plastic toy he makes. If he makes 40 wooden toys and 20 plastic toys. How much would he receive altogether?

QUESTION 3

SOLVE PROBLEMS INVOLVING TAXATION

a) Andrew earns an annual gross salary of $47 560. His employer deducts $15 000 in tax, $550 in union fees and $6 543.67 for superannuation per year. What is his annual net income?

b) The tax table below shows the tax rates for 2016.

<table>
<thead>
<tr>
<th>TAXABLE INCOME</th>
<th>TAX PAYABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - $18 200</td>
<td>Nil</td>
</tr>
<tr>
<td>$18 201 - $37 000</td>
<td>19c for each $1 over $18 200</td>
</tr>
<tr>
<td>$37 001 - $80 000</td>
<td>$3 572 plus 32.5c for each $1 over $37 000</td>
</tr>
<tr>
<td>$80 001 - $180 000</td>
<td>$17 547 plus 37c for each $1 over $80 000</td>
</tr>
<tr>
<td>$180 001 and over</td>
<td>$54 547 plus 45c for each $1 over $180 000</td>
</tr>
</tbody>
</table>

i) Audrey earns a salary of $46 540. How much tax does she pay?
ii) Audrey’s employer has already taken out $7750 in PAYG tax instalments this year. Is she entitled to a refund or is she liable to pay tax back? What is her refund or liability amount?

iii) Audrey is required to pay a Medicare levy of 1.5% on her taxable income. Calculate the Medicare Levy she is required to pay.

QUESTION 4

a) Find the simple interest earned on an investment of $6500 at 6% p.a. for 4 years.

b) Rob borrowed $3540 for 3 years and was charged simple interest at 2.5% p.a. How much did he repay altogether including interest?

c) Mary took out a student loan of $1500 for 7 months at 7.5% p.a. Calculate the simple interest on her loan.

d) Fadila purchased a television for her room for $5400. She paid an initial deposit of $1500 and borrowed the rest at 12.8% p.a. simple interest for 2 years. Calculate:
   i) the amount borrowed.
   ii) the simple interest charged on the amount borrowed.
iii) the total to be repaid.

iv) the total price paid for the television.

---

**QUESTION 5**

**CALCULATE COMPOUND INTEREST USING REPEATED APPLICATIONS OF SIMPLE INTEREST**

a) Lisa invested $3000 at 7% p.a. compounded over 3 years.

Calculate:

i) the total of her investment at the end of 3 years.

---

ii) the compound interest earned.

---

b) Use the repeated multiplication method to find the final value of $5400 invested at 6.3% p.a. for 3 years.

---
c) **COMPOUND INTEREST TABLE**

<table>
<thead>
<tr>
<th>NUMBER OF PERIODS (N)</th>
<th>INTEREST RATE PER COMPOUNDING PERIOD (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>2</td>
<td>1.0404</td>
</tr>
<tr>
<td>4</td>
<td>1.0824</td>
</tr>
<tr>
<td>6</td>
<td>1.1262</td>
</tr>
</tbody>
</table>

Use the compound interest table above to calculate the total investment for the following:

i) $5400 invested at 3% p.a. compounded yearly for 2 years.

ii) $4500 invested at 6% p.a. compounded half yearly for 3 years.

d) Compare the following investments. Which is the better investment? (show working)

**INVESTMENT A**
$5000 invested at a simple interest rate at 3% p.a. for 6 years

**INVESTMENT B**
$5000 invested at 3% p.a. Compounded yearly for 6 years (Use the compound interest table)
Mathematics Stage 5.1: Indices

Name: _____________________________________________
Class: _____________________________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

STUDENTS HAVE LEARNT TO

☐ Use index notation for positive integral indices
☐ Express a whole number as a product of its prime factors
☐ Apply the order of operations to evaluate indices
☐ Determine and apply tests of divisibility for 2, 3, 4, 5, 6, 10
☐ Determine and apply the index laws for numerical expressions and the meaning of the zero index
☐ Verify the index laws using a calculator

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

NUMBER AND ALGEBRA: INDICES

STAGE 5.1: INDICES

SYLLABUS REFERENCE

MA5.1-5NA - A student operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases.

QUESTION 1

APPLY THE INDEX LAWS TO SIMPLIFY ALGEBRAIC EXPRESSIONS WHEN MULTIPLYING INDICES

a) Write the following in index notation:
   i) \(a \times a \times a \times a = \) ____________________________
   ii) \(2 \times 2 \times 2 \times 2 \times 2 = \) __________________________
b) Simplify the following:

i) \(2^3 \times 2^2 = \)

ii) \(m^2 \times m^3 \times m^4 = \)

iii) \(2x^2 \times 3x^4 = \)

iv) \(-p^2 \times (-5p^2) = \)

v) \(4xy^2 \times 8x^2 y^3 = \)

---

**QUESTION 2**

APPLY INDEX LAWS WHEN DIVIDING INDICES

Simplify the following:

a) \(y^5 \div y^2 = \)

b) \(8x^2 \div 4x = \)

c) \(30k^3 \div -5k^2 = \)

d) \(\frac{24x^3y^5}{6x^2y^3} = \)

---

**QUESTION 3**

APPLY INDEX LAWS WHEN DIVIDING INDICES

Simplify the following, giving answers in index form:

a) \((5^2)^2 = \)

b) \((2q^3)^3 = \)

c) \(\frac{(7c^2)}{(d)} = \)

---

**QUESTION 4**

THE ZERO INDEX

Simplify the following:

a) \(x^0 = \)

b) \(2x^0 = \)

c) \((-5p)^0 = \)
STAGE 5.1: INDICES

QUESTION 5

DEFINE AND USE NEGATIVE-INTEGER INDICES TO EVALUATE NUMERICAL EXPRESSIONS

a) Rewrite the following with a positive index:

i) \(2^{-2} = \) ____________________________

ii) \(5^{-3} = \) ____________________________

b) Write the following with negative indices:

i) \(\frac{1}{2^4} = \) ____________________________

ii) \(-\frac{1}{4^3} = \) ____________________________

c) Evaluate the following: (giving your answers as fractions)

i) \(3^{-2} = \) ____________________________

ii) \(-5^{-3} = \) ____________________________

iii) \(-\frac{1}{4^2} + 2^{-2} = \) ____________________________
Mathematics Stage 5.1: Linear Relationships

Name: ____________________________

Class: ____________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

STUDENTS HAVE LEARNT TO

☐ Locate and describe points on the Cartesian plane using coordinates
☐ Describe translations and reflections in an axis on the Cartesian plane
☐ Describe rotations of multiples of 90° on the Cartesian plane
☐ Recognise, describe and record geometric and number patterns in words and algebraic symbols
☐ Plot linear relationships created from simple patterns and equations
☐ Solve simple linear equations using graphical techniques

SYLLABUS REFERENCE

MA5.1-6NA - A student determines the midpoint, gradient and length of an interval, and graphs linear relationships.
STAGE 5.1: LINEAR RELATIONSHIPS

QUESTION 1
FIND THE MIDPOINT, GRADIENT AND LENGTH OF INTERVALS ON THE CARTESIAN PLANE USING INFORMAL STRATEGIES

a) Does the interval PQ have a positive or negative gradient? ____________________________

b) Determine the midpoint, gradient and length for the interval joining P (-1,-1) and Q (4,3) on the Cartesian plane.

i) Midpoint ____________________________

ii) Gradient ____________________________

iii) Length ____________________________

QUESTION 2
GRAPH LINEAR RELATIONSHIPS FROM EQUATIONS

Graph the following linear relationships on the Cartesian plane.

a) \( x = 2 \)

\[
\begin{array}{cccccc}
  x & & & & & \\
  y & -2 & -1 & 0 & 1 & 2 & 3 \\
\end{array}
\]

b) \( y = -2 \)

\[
\begin{array}{cccccc}
  x & -2 & -1 & 0 & 1 & 2 & 3 \\
  y & & & & & & \\
\end{array}
\]
**STAGE 5.1: LINEAR RELATIONSHIPS**

**QUESTION 3**

**GRAPH LINEAR RELATIONSHIPS FROM EQUATIONS**

For the linear equation \( y = x + 3 \):

a) identify the \( x \) and \( y \) intercepts.

b) graph the linear equation \( y = x + 3 \).

![Graph of \( y = x + 3 \)]

**QUESTION 4**

**DETERMINE WHETHER A POINT LIES ON A LINE BY SUBSTITUTION**

Does the point (2,3) lie on the line \( y = 3x - 2 \). (show working)

**QUESTION 5**

**DETERMINE THAT PARALLEL LINES HAVE EQUAL GRADIENTS**

From the list below circle the lines that are parallel.

\( y = 3x \) \hspace{1cm} \( y = 2x + 3 \) \hspace{1cm} \( y = 3x + 1 \) \hspace{1cm} \( y = -3x \)
**Stage 5.1: Non-Linear Relationships**

Name: 

Class: 

---

**STAGE 5.1: NON-LINEAR RELATIONSHIPS**

**SYLLABUS REFERENCE**

**MA5.1-7NA** - A student graphs simple non-linear relationships.

---

**QUESTION 1**

**GRAPH SIMPLE PARABOLAS AND EXPONENTIALS ON THE CARTESIAN PLANE USING TABLES OF VALUES**

Graph the following non-linear relationships on the Cartesian plane.

i) \( y = x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii) \( y = 2^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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STAGE 5.1: NON-LINEAR RELATIONSHIPS

QUESTION 2

CONNECT THE SHAPE OF A NON-LINEAR GRAPH WITH THE DISTINGUISHING FEATURES OF ITS EQUATION

Match the following equations to the graphs A, B and C.

i) \(x^2 + y^2 = 4\)

ii) \(y = x^2 + 3\)

iii) \(x^2 + y^2 = 25\)

QUESTION 3

COMPARE NON-LINEAR RELATIONSHIPS

i) Sketch the graph \(y = x^2 + 1\) and \(y = x^2 - 1\) on the same number plane. Comment on where the graphs cross the y axis.

\[
\begin{array}{c|c|c|c|c|c}
\text{x} & -2 & -1 & 0 & 1 & 2 \\
\text{y} & & & & & \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\text{x} & -2 & -1 & 0 & 1 & 2 \\
\text{y} & & & & & \\
\end{array}
\]

ii) What effect does the constant have on each of the graphs?
Mathematics Stage 5.1: Area and Surface Area

Name: ________________________________

Class: ________________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

MEASUREMENT AND GEOMETRY: AREA AND SURFACE AREA

STUDENTS HAVE LEARNT TO

☐ Convert between metric units of area
☐ Establish and use formulas to find the areas of triangles, special quadrilaterals and circles
☐ Solve problems involving area

SYLLABUS REFERENCE

MA5.1-8MG - A student calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms.

STAGE 5.1: AREA AND SURFACE AREA

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

QUESTION 1

SOLVE PROBLEMS INVOLVING AREAS OF COMPOSITE SHAPES BY DISSECTION INTO TRIANGLES, QUADRILATERALS, QUADRANTS, SEMI CIRCLES AND SECTORS

Calculate the area of the following:

a) 

b) 

10 cm

7 cm

10 cm

6.3 cm

18.4 cm
STAGE 5.1: AREA AND SURFACE AREA

c) calculate the shaded area.

d) calculate the shaded area.

e) calculate the shaded area.

f) calculate the shaded area.
STAGE 5.1: AREA AND SURFACE AREA

QUESTION 2  SOLVE PROBLEMS INVOLVING THE SURFACE AREA OF RECTANGULAR AND TRIANGULAR PRISMS

Calculate the surface area of the following:

a)  

b)  

(Hint you will need to use Pythagoras’ Theorem)

QUESTION 3  SOLVE PROBLEMS INVOLVING THE SURFACE AREA OF RECTANGULAR AND TRIANGULAR PRISM

The diagram below shows a box which is open at the top:

a) How many faces does the box have?

b) Calculate the surface area of the box.
Mathematics Stage 5.1: Numbers of Any Magnitude

Name: __________________________________________

Class: __________________________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

MEASUREMENT AND GEOMETRY: NUMBERS OF ANY MAGNITUDE

STUDENTS HAVE LEARNT TO

- Perform operations with time units using mental computations and with a calculator
- Interpret international time zones

SYLLABUS REFERENCE

MA5.1-9MG - A student interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures.

STAGE 5.1: NUMBERS OF ANY MAGNITUDE

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

QUESTION 1

ROUND NUMBERS TO A SPECIFIED NUMBER OF SIGNIFICANT FIGURES

Round the following to 3 significant figures:

a) 2432
b) 0.237541
c) 1.287852
STAGE 5.1: NUMBERS OF ANY MAGNITUDE

QUESTION 2

CONVERT BETWEEN METRIC UNITS FOR VERY LARGE AND VERY SMALL MEASUREMENTS, INCLUDING MEASUREMENTS OF DIGITAL INFORMATION

a) Convert the following:
   i) 0.0078 m to cm
   ii) 0.00234 mm to m
   iii) 1860 nm to m
   iv) 0.006 cm to μm

b) Convert the following:
   i) 7850 000 mL to L
   ii) 0.00007826 kL to mL

c) Convert the following:
   i) 7 MB to KB
   ii) 5200 GB to TB
   iii) 2.1 TB to KB

d) Convert the following:
   i) 7.8 minutes to milliseconds
   ii) 170000 microseconds to minutes
   iii) 1 fortnight to seconds
   iv) 897 years to decades

QUESTION 3

EXPRESS NUMBERS IN SCIENTIFIC NOTATION USING POSITIVE AND NEGATIVE POWERS OF 10

a) Express the following in scientific notation:
   i) 2800
   ii) 1 240 000
   iii) 0.03456
   iv) 0.0000541
STAGE 5.1: NUMBERS OF ANY MAGNITUDE

b) Express each number in decimal form:

i) \(2.6 \times 10^2\)

ii) \(4.56 \times 10^{-4}\)

QUESTION 4 ORDER NUMBERS EXPRESSED IN SCIENTIFIC NOTATION

Write each set of numbers in ascending order.

a) \(1.2 \times 10^9, 6.3 \times 10^8, 3.5 \times 10^9\)

b) \(4.1 \times 10^{-2}, 3.8 \times 10^{-1}, 4.8 \times 10^{-3}\)

QUESTION 5 SOLVE PROBLEMS INVOLVING SCIENTIFIC NOTATION

a) The Earth is \(1.52 \times 10^8\) kilometres from the Sun. If the speed of light is \(3 \times 10^5\) kilometres per second, how long does it take light to travel from the Sun to Earth? Give your answer to the nearest second.

b) Rachael is an author. She types 28,000 words every day. How many words will she type in a year? (answer in scientific notation)

QUESTION 6 LIMITS OF ACCURACY

Lisa weighs her dog and records its weight as 16kg. What:

a) is the limit of accuracy of Lisa’s measurement?

b) could be the actual weight of Lisa’s dog?

QUESTION 7 DEGREE OF ACCURACY

Round the following to the appropriate degree of accuracy.

a) \(26.1 + 13.45 = \) 

b) \(140 + 18.1 = \)
Mathematics Stage 5.1: Right-Angled Triangles (Trigonometry)

Name: 

Class: 

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

MEASUREMENT AND GEOMETRY: TRIGONOMETRY

STUDENTS HAVE LEARNT TO

☐ Establish and apply Pythagoras’ theorem to find sides in right-angled triangles
☐ Solve problems involving Pythagoras’ theorem

SYLLABUS REFERENCE

MA5.1-10MG - A student applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression

STAGE 5.1: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 1

USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (ANGLES MEASURED TO NEAREST DEGREE)

a) In ΔABC identify the following sides with respect to θ:
   i) hypotenuse 
   ii) adjacent side 
   iii) opposite side 

b) Find the value for sin θ, cos θ and tan θ in ΔDEF:
   i) sin θ 
   ii) cos θ 
   iii) tan θ 

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c) Evaluate the following to 2 decimal places:
   i) $\cos 73^\circ =$ .................................................................
   ii) $\tan 12^\circ =$ .................................................................
   iii) $3 \times \sin 23^\circ =$ .................................................................

d) Find $\theta$ to the nearest degree:
   i) $\cos \theta = 0.143$ .................................................................
   ii) $\tan \theta = 1.56$ .................................................................
   iii) $\sin \theta = \frac{3}{5}$ .................................................................

---

**QUESTION 2**

**USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES**  
(ANGLES MEASURED TO NEAREST DEGREE)

a) Find the value of the pronumeral in the following:

i) ![Diagram 1](#) .................................................................

ii) ![Diagram 2](#) .................................................................
b) Find the value of \( \theta \) in the following to the nearest degree:

i) 
\[
\begin{align*}
\triangle & \quad \text{3} \\
& \quad \text{4} \\
\theta & \quad \text{520 m}
\end{align*}
\]

ii) 
\[
\begin{align*}
\text{6 cm} & \quad \theta \\
& \quad \text{12 cm}
\end{align*}
\]

c) The angle of depression of a boat from the top of the cliff is 54°. If the boat is 520 m from the base of the cliff, calculate the height of the cliff? (answer to the nearest metre)

\[
\begin{align*}
\text{Cliff} & \quad 54^\circ \\
& \quad 520 \text{ m}
\end{align*}
\]
Mathematics Stage 5.1: Properties of Geometric Figures

Name: ____________________________________________

Class: ____________________________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

STUDENTS HAVE LEARNT TO

☐ Classify and determine properties of triangles and quadrilaterals
☐ Identify line and rotational symmetries
☐ Determine the angle sums of triangles and quadrilaterals
☐ Use properties of shapes to find unknown sides and angles in triangles and quadrilaterals giving a reason
☐ Identify congruent figures
☐ Identify congruent triangles using the four tests

MEASUREMENT AND GEOMETRY: PROPERTIES OF GEOMETRIC FIGURES

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.1: PROPERTIES OF GEOMETRIC FIGURES

SYLLABUS REFERENCE

MA5.1-11MG - A student describes and applies the properties of similar figures and scale drawings.

QUESTION 1

DESCRIBE SIMILAR FIGURES

Explain what makes two shapes similar. ____________________________________________

__________________________________________
STAGE 5.1: PROPERTIES OF GEOMETRIC FIGURES

QUESTION 2
IDENTIFY SIMILAR FIGURES AND DESCRIBE THEIR PROPERTIES INFORMALLY

Which two pairs of triangles below are similar?

How do you know?

\[80\degree, 40\degree, 70\degree, 70\degree, 80\degree, 40\degree\]

QUESTION 3
IDENTIFY SIMILAR FIGURES AND DESCRIBE THEIR PROPERTIES INFORMALLY

Using the two similar trapeziums below:

a) name the four pairs of matching sides.

b) name the four pairs of matching angles.
STAGE 5.1: PROPERTIES OF GEOMETRIC FIGURES

QUESTION 4

USE SCALE FACTORS AND PROPORTION STATEMENTS TO DETERMINE UNKNOWN LENGTHS IN SIMILAR FIGURES

a) Find the scale factor for the similar figures below:

i) 

ii) 

b) For the similar figures below find:

i) the scale factor

ii) the length of the unknown side

QUESTION 5

USE SCALES ON MAPS, PLANS AND DRAWINGS

James has a small photograph of a door that he wants to enlarge to have a length of 34 cm.

i) What is the scale used?

ii) Hence, find the real width of the door.
STAGE 5.1: PROPERTIES OF GEOMETRIC FIGURES

QUESTION 6 CONSTRUCT SCALE DRAWINGS

Draw the image of the figures below using the given scale factor.

a) Scale factor = 2

b) Scale factor = $\frac{1}{3}$
Mathematics Stage 5.1: Single Variable Data Analysis

Name: ____________________________

Class: ____________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

STATISTICS AND PROBABILITY: SINGLE VARIABLE DATA ANALYSIS

STUDENTS HAVE LEARNT TO

☐ Calculate mean, median, mode and range for sets of data
☐ Investigate the effect of outliers on the mean and median
☐ Describe and interpret a variety of data displays using mean, median and range
☐ Calculate and compare summary statistics of different samples drawn from the same population

SYLLABUS REFERENCE

MA5.1-12SP - A student uses statistical displays to compare sets of data, and evaluates statistical claims made in the media.

QUESTION 1

DESCRIBE DATA, USING TERMS INCLUDING ‘SKEWED’, ‘SYMMETRIC’ AND ‘BI-MODAL’

Describe the following data representations as positively skewed, negatively skewed, symmetric or bi-modal.

[Graphs of different data distributions]
STAGE 5.1: SINGLE VARIABLE DATA ANALYSIS

QUESTION 2 CONSTRUCT AND INTERPRET BACK-TO-BACK STEM AND LEAF PLOTS

The results of a mathematics exam are given below for a class of year 9 students:

<table>
<thead>
<tr>
<th>GIRLS</th>
<th>9</th>
<th>22</th>
<th>24</th>
<th>25</th>
<th>33</th>
<th>36</th>
<th>37</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOYS</td>
<td>5</td>
<td>6</td>
<td>12</td>
<td>25</td>
<td>26</td>
<td>33</td>
<td>34</td>
<td>35</td>
</tr>
</tbody>
</table>

i) Complete the following back to back stem and leaf plot to display the above results.

<table>
<thead>
<tr>
<th>LEAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

ii) Determine whether a boy or a girl achieved the lowest score?

iii) Use the mean, median and range to compare the two data sets. Who achieved the better results overall, boys or girls?

iv) Comment on the skewness of the data for the boys’ results.

QUESTION 3 INTERPRET AND CRITICALLY EVALUATE REPORTS IN THE MEDIA AND ELSEWHERE THAT LINK CLAIMS TO DATA DISPLAYS AND STATISTICS

Interpret and comment on the following statistical statements made in the media.

i) Data collected about Facebook and Twitter states that 25% of users on social media do not bother with privacy settings.

ii) According to the 2006 Census, about 36% of Ultimo residents walked to work.
Mathematics Stage 5.1: Probability

Name: ____________________________
Class: ____________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

STATISTICS AND PROBABILITY: PROBABILITY

STUDENTS HAVE LEARNT TO

☐ Construct sample spaces for single-step experiments with equally likely outcomes
☐ Find probabilities of events in single-step experiments
☐ Identify complementary events and use the sum of probabilities to solve problems
☐ Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’
☐ Represent events in two-way tables and Venn diagrams and solve related problems

SYLLABUS REFERENCE

MA5.1-12SP - A student uses statistical displays to compare sets of data, and evaluates statistical claims made in the media.

QUESTION 1

CALCULATE RELATIVE FREQUENCIES TO ESTIMATE PROBABILITIES

The following frequency table shows the results of data collected from 50 people on how they travel to work.

<table>
<thead>
<tr>
<th>TRANSPORT</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>4</td>
</tr>
<tr>
<td>Train</td>
<td>20</td>
</tr>
<tr>
<td>Bus</td>
<td>16</td>
</tr>
<tr>
<td>Ferry</td>
<td>5</td>
</tr>
<tr>
<td>Walk</td>
<td>2</td>
</tr>
<tr>
<td>Bicycle</td>
<td>3</td>
</tr>
</tbody>
</table>

i) Calculate the relative frequency for travelling by car.

ii) What is the relative frequency for travelling by train or by ferry?
STAGE 5.1: PROBABILITY

QUESTION 2

PREDICT THE PROBABILITY OF AN EVENT FROM EXPERIMENTAL DATA USING RELATIVE FREQUENCIES

The spinner shown below was spun 150 times and the results are shown in the table.

![Spinner Diagram]

<table>
<thead>
<tr>
<th>OUTCOME</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>28</td>
</tr>
<tr>
<td>Green</td>
<td>23</td>
</tr>
<tr>
<td>Yellow</td>
<td>24</td>
</tr>
<tr>
<td>Purple</td>
<td>21</td>
</tr>
<tr>
<td>Red</td>
<td>54</td>
</tr>
</tbody>
</table>

If the spinner was spun 500 times, how many times would you expect it to come up red? ____________

QUESTION 3

CALCULATE PROBABILITIES FROM VENN DIAGRAMS AND TWO-WAY TABLES

a) The Venn diagram shows the sports that are played in year 9.

i) How many students are there altogether? ____________

ii) How many students do not play sport? ____________

iii) What is the probability of choosing a student who plays both tennis and soccer? ____________

iv) What is the probability of choosing a student who plays tennis or soccer? ____________

<table>
<thead>
<tr>
<th>Sports played in year 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tennis</td>
</tr>
<tr>
<td>Soccer</td>
</tr>
<tr>
<td>Tennis</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>Soccer</td>
</tr>
<tr>
<td>42</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
b) The two way table below shows the employment status for Luke Town by gender.

<table>
<thead>
<tr>
<th>EMPLOYED STATUS</th>
<th>EMPLOYED</th>
<th>UNEMPLOYED</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEMALE</td>
<td>320</td>
<td>54</td>
</tr>
<tr>
<td>MALE</td>
<td>430</td>
<td>36</td>
</tr>
</tbody>
</table>

| TOTAL           | 750      | 90         | 840        |

i) How many females were unemployed?  

ii) What is the probability of choosing a male or female who is unemployed?  

iii) What is the probability of randomly choosing someone who is male and employed?
MATHEMATICS
STAGE 5.2
Mathematics Stage 5.2: Financial Mathematics

Name: 
Class: 

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

NUMBER AND ALGEBRA: FINANCIAL MATHEMATICS

STUDENTS HAVE LEARNT TO

☐ Solve problems involving earning money and taxation
☐ Apply the simple interest formula to solve problems, including buying on terms
☐ Calculate compound interest using repeated applications of simple interest

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.2: FINANCIAL MATHEMATICS

SYLLABUS REFERENCE

MA5.2-4NA - A student solves financial problems involving compound interest.

QUESTION 1

APPLY THE COMPOUND INTEREST FORMULA TO SOLVE PROBLEMS

(a) i) Use the compound interest formula to determine the value of an investment of $5000 compounded yearly at 4% p.a. at the end of 3 years.

ii) Calculate the amount of interest gained on the investment.
b) Using the information below:

<table>
<thead>
<tr>
<th>TYPE A</th>
<th>TYPE B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10000 compounded yearly at 5% p.a for 5 years</td>
<td>$10000 compounded monthly at 5% p.a for 5 years</td>
</tr>
</tbody>
</table>

determine which is the best investment. *(show working)*

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________

c) An investment is worth $10000 at the end of 3 years compounded annually. Calculate the interest rate that was applied if the initial principal amount was $6000. *(answer to 1 decimal place)*

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________

d) Lisa received a bonus of $7500 at the end of the year. She needs a total of $10000 to go on her dream holiday. Use the guess and refine method to determine how long it will take to reach her goal if she invests her bonus at 7% p.a. compounded yearly.

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________
QUESTION 2

SOLVE PROBLEMS INVOLVING DEPRECIATION

a) A computer depreciates in value at 15% p.a. If the computer was bought for $2300, how much is it worth after 3 years?

b) A Toyota Corolla was bought for $23 000 and depreciates at a rate of 21% p.a.

i) Calculate the value of the car at the end of 3 years?

ii) How much did the car depreciate in value?
Mathematics Stage 5.2: Rates and Ratios

Name: ____________________________________________________________

Class: ____________________________________________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

NUMBER AND ALGEBRA: RATES AND RATIOS

STUDENTS HAVE LEARNT TO

☐ Simplify ratios
☐ Solve problems involving ratios
☐ Convert into simplified rate
☐ Solve problems involving rates
☐ Interpret and draw distance/time graphs

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.2: RATES AND RATIOS

SYLLABUS REFERENCE

MA5.2-5NA - A student recognises direct and indirect proportion, and solves problems involving direct proportion.

QUESTION 1

Convert the following:

a) $5.50/h to $/min

b) 250 L/h to mL/h

c) 65 km/h to m/s
STAGE 5.2: RATES AND RATIOS

QUESTION 2
SOLVE PROBLEMS INVOLVING CONSTANT RATES OF CHANGE

a) Calculate the average speed of a car that travels 250 km in 5 hours.

b) A tap that is leaking has filled a 2.5 L bucket in 6 hours. At what rate was the tap leaking per hour (in mL/h)?

c) A truck used 45 L of petrol in 300 km. Express the fuel consumption in litres per 100 km.

QUESTION 3
IDENTIFY EXAMPLES OF DIRECT AND INVERSE PROPORTION

Classify the following as inverse (indirect) or direct proportion.

a) As the number of hours at work increases, the earnings also increase.

b) As speed increases, the time taken to travel a particular distance decreases.

c) As the size of the airplane increases, the cost to construct the airplane increases.
STAGE 5.2: RATES AND RATIOS

QUESTION 4

INTERPRET GRAPHS OF CHANGE WHERE THE RELATIONSHIP BETWEEN THE VARIABLES IS CONSTANT

Interpret this temperature conversion graph and answer the following questions:

a) convert 20°C to °F.

b) convert 100°C to °F.

c) is the gradient of the graph positive or negative?

d) are the variables in direct linear proportion? Explain your answer.
STAGE 5.2: RATES AND RATIOS

QUESTION 5  CALCULATE THE CONSTANT OF PROPORTIONALITY

Sue’s fuel usage \( (F) \) is in direct linear proportion with the distance that she travels \( (d) \).

a) Establish an equation that can be used to calculate the amount of fuel she uses on her trips.

b) Calculate the constant of proportionality if she used 25 L while travelling 200 km.

c) What distance could she travel on 40 L of fuel?

d) Construct a graph to model Sue’s fuel usage.

Sue’s Fuel Consumption

<table>
<thead>
<tr>
<th>Fuel used (L)</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>200</td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

0 50 100 150 200 250 300 350

0 10 20 30 40

Distance (km)

Fuel used (L)
Mathematics Stage 5.2: Algebraic Techniques

Name: ____________________________________________

Class: ____________________________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

NUMBER AND ALGEBRA: ALGEBRAIC TECHNIQUES

STUDENTS HAVE LEARNT TO

☐ Use letters to represent numbers
☐ Recognise and use simple equivalent algebraic expressions
☐ Simplify algebraic expressions involving the four operations
☐ Substitute into algebraic expressions
☐ Expand and factorise simple algebraic expressions

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.2: ALGEBRAIC TECHNIQUES

SYLLABUS REFERENCE

MA5.2-6NA - A student simplifies algebraic fractions, expands and factorises quadratic expressions.

QUESTION 1

APPLY THE FOUR OPERATIONS TO SIMPLIFY SIMPLE ALGEBRAIC FRACTIONS

a) Simplify the following:

i) \( \frac{x}{2} + \frac{x}{3} \)  

ii) \( \frac{4y}{5} - \frac{2y}{3} \)
STAGE 5.2: ALGEBRAIC TECHNIQUES

iii) \( \frac{2a}{3} \times \frac{3a}{8} \) 

iv) \( \frac{x^2 + y}{5} \div \frac{y}{15} \)

v) \( \frac{6}{n} - \frac{1}{2n} \)

b) Expand and simplify:

i) \(2y(y - 3) + 3(2y + 4)\) 

ii) \(4a(10a - 7) - (3a - 4)\)

iii) \((x + 2)(x + 4)\) 

iv) \((k - 3)(2k + 5)\)

QUESTION 2

a) Factorise the following:

i) \(4d^3 - 4d\) 

ii) \(21xy - 3x + 9x^2\) 

iii) \(h^2 + 8h + 12\)
Mathematics Stage 5.2: Indices

Name: 

Class: 

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

NUMBER AND ALGEBRA: INDICES

STUDENTS HAVE LEARNT TO

☐ Apply the index laws to simplify algebraic expressions with positive-integer indices and the zero index
☐ Define and use negative-integer indices to evaluate numerical expressions

SYLLABUS REFERENCE

MA5.2-7NA - A student applies index laws to operate with algebraic expressions involving integer indices.

STAGE 5.2: INDICES

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

WRITE EXPRESSIONS INVOLVING NEGATIVE INDICES AS EXPRESSIONS INVOLVING POSITIVE INDICES, AND VICE VERSA

QUESTION 1

a) Write the following with a negative index:
   i) \( \frac{1}{x^2} \)
   ii) \( \frac{2}{m^3} \)
   iii) \( \frac{1}{4a^2} \)

b) Write the following with positive indices:
   i) \( x^{-3} \)
   ii) \( 4m^{-6} \)

c) By substituting \( x = 1 \), show whether \( x^{-3} \) is equivalent to \(-3x\). 


STAGE 5.2: INDICES

QUESTION 2

a) Simplify the following:
   i) $2m^6 \times 3m^{-4}$
   ii) $(x^3)^{-2}$
   iii) $3y^{-4} \times y^7$
   iv) $15k^7 \div 5k^{-3}$
   v) $(4m^{-2})^3$

b) State whether the following are true or false:
   i) $4x^0 = 1$
   ii) $a^6 \div a^4 = a^2$
   iii) $(6m)^0 = 1$
   iv) $3x^5 \div 3x^5 = x$
   v) $4c^{-3} = \frac{1}{4c^3}$
   vi) $12m^{10} \times 2m^{-2} = 24m^8$

QUESTION 3

Simplify the following:

a) $-3m^2 (5m^2 + 2m^4n)$

b) $\frac{16x^2}{4x}$
### STAGE 5.2: INDICES

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>c)</td>
<td>( \frac{20a^3b^2}{15ab^3} )</td>
<td>d)</td>
<td>( \frac{3xy}{2} \times \frac{2x}{6} )</td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td>( \frac{b^2c^3}{4} \times \frac{12}{bc^2} )</td>
<td>f)</td>
<td>( \frac{4n^3}{3m^2} \div \frac{12n}{3m^3} )</td>
<td></td>
</tr>
</tbody>
</table>
Mathematics Stage 5.2: Equations

Name: ___________________________  
Class: ___________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

NUMBER AND ALGEBRA: EQUATIONS

STUDENTS HAVE LEARNT TO

☐ Solve simple linear equations using algebraic techniques
☐ Solve simple quadratic equations of the form $x^2 = c$

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.2: EQUATIONS

SYLLABUS REFERENCE

MA5.2-8NA - A student solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques.

QUESTION 1

SOLVE LINEAR EQUATIONS INVOLVING GROUPING SYMBOLS

Solve the following:

a) $2(y + 4) + 3(y - 2) = 10$

b) $3(2y + 5) = 4y - 9$

c) $4(2k - 6) = -3(2k + 3)$
STAGE 5.2: EQUATIONS

QUESTION 2

SOLVE LINEAR EQUATIONS INVOLVING ALGEBRAIC FRACTIONS

Solve the following:

a) \[ \frac{3y + 4}{3} = 5 \]

b) \[ \frac{y - 2}{4} + 5 = 7 \]

c) \[ \frac{3x}{5} - 4 = -2 \]

d) \[ \frac{2m}{2} + \frac{m}{3} = 4 \]

e) \[ \frac{7y + 2}{4} = \frac{2y + 5}{2} \]
STAGE 5.2: EQUATIONS

QUESTION 3

SOLVE QUADRATIC EQUATIONS OF THE FORM AX^2 = C

a) Solve the following, leave answers to 1 decimal place where necessary:

i) \( y^2 = 4 \)  ________________________________

ii) \( 2m^2 = 32 \)  ________________________________

iii) \(-5x^2 = -40 \)  ________________________________

b) Solve the following, leaving answers in exact form:

i) \( k^2 = 20 \)  ________________________________

ii) \( 6k^2 = 4 \)  ________________________________

QUESTION 4

SOLVE QUADRATIC EQUATIONS OF THE FORM AX^2 + BX + C = 0 (WHERE A = 1) USING FACTORS

Solve the following quadratic equations:

i) \( x^2 + 2x + 1 = 0 \)  ________________________________

ii) \( y^2 - 5y + 6 = 0 \)  ________________________________

iii) \( z^2 - 9z - 10 = 0 \)  ________________________________

QUESTION 5

SOLVE EQUATIONS RESULTING FROM SUBSTITUTION INTO FORMULAS

a) The perimeter of a rectangle is given by \( P = 2L + 2B \), find:

i) \( B \): if \( L = 5 \) and \( P = 25 \)  ________________________________

ii) \( L \): if \( P = 40 \) and \( B = 4 \)  ________________________________
b) The formula for converting temperature recorded in °F to temperature in °C, is \( C = \frac{5}{9}(F - 32) \).
Express 80°F in °C, correct to the nearest degree.

---

**QUESTION 6**

**SOLVE WORD PROBLEMS USING LINEAR EQUATIONS**

a) The sum of three consecutive integers is 42. What are the three integers?

---

b) The area of a triangle with base 10 cm is 45 cm². Find the perpendicular height of the triangle.
STAGE 5.2: EQUATIONS

QUESTION 7

SOLVE LINEAR INEQUALITIES AND GRAPH THE SOLUTIONS

Solve the following inequalities and then graph the solutions on a number line.

a) $2x - 1 > 9$

b) $10 < b - 5$

c) $3(b - 2) \geq 18$

d) $\frac{y + 5}{3} < -2$

e) $2 - 5m \leq 6$
STAGE 5.2: EQUATIONS

QUESTION 8

SOLVE LINEAR SIMULTANEOUS EQUATIONS
USING ALGEBRAIC AND GRAPHICAL TECHNIQUES

a) On the same set of axis, graph \( y = 9 - 2x \) and \( y = 4x - 6 \), then solve simultaneously.

b) Solve the following equations simultaneously using the substitution method.

\[
\begin{align*}
\text{y} &= x + 4 \\
\text{y} &= 3x - 2
\end{align*}
\]

c) Solve the following equations simultaneously using the elimination method.

\[
\begin{align*}
\text{x} + 3\text{y} &= 7 \\
4\text{x} - 3\text{y} &= 13
\end{align*}
\]
Mathematics Stage 5.2: Linear Relationships

Name: ____________________________________________

Class: __________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

NUMBER AND ALGEBRA: LINEAR RELATIONSHIPS

STUDENTS HAVE LEARNT TO

☐ Find the midpoint, gradient and length of intervals on the Cartesian plane using informal strategies

☐ Graph linear relationships from equations

☐ Determine that parallel lines on the Cartesian plane have equal gradients

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

MA5.2-9NA - A student uses the gradient-intercept form to interpret and graph linear relationships.

STAGE 5.2: LINEAR RELATIONSHIPS

SYLLABUS REFERENCE

QUESTION 1

APPLY THE GRADIENT-INTERCEPT FORM OF THE EQUATION OF A STRAIGHT LINE TO INTERPRET AND GRAPH STRAIGHT LINES

a) Find the gradient and y intercept of the following linear equations:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Gradient</th>
<th>Y intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) $y = 3x - 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ii) $y = 5 - 4x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii) $6y - 3x = 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) $2x - 2y + 3 = 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
STAGE 5.2: LINEAR RELATIONSHIPS

b) Write the equation of a straight line with gradient $m = -2$ and $y$ intercept of 6.

c) Graph the following line by first determining the gradient and $y$ intercept of the equation:

$$y = 2 - 4x$$

d) Determine the following for the given line:

i) the $y$ intercept. ________________

ii) the gradient. ________________

iii) the equation of the line. ________________
STAGE 5.2: LINEAR RELATIONSHIPS

QUESTION 2

APPLY THE PROPERTIES OF THE GRADIENTS OF PARALLEL AND PERPENDICULAR LINES ON THE CARTESIAN PLANE

a) Circle which of the following lines are perpendicular to each other.

(i) \( y = -2x + 2 \)  
(ii) \( y = -3x + 2 \)  
(iii) \( y = \frac{1}{2}x + 4 \)  
(iv) \( y = -\frac{1}{2}x + 4 \)

b) Determine the equation of a line with y-intercept at -2 that is:

i) parallel to \( y = 3 - 4x \).

ii) perpendicular to the line \( 2x - 3y + 4 = 0 \).

iii) parallel to \( y = 3 - 4x \) and y-intercept at -2.

iv) perpendicular to the line \( 2x - 3y + 4 = 0 \) and y-intercept at -2.

c) Determine the equation of the following two lines and show that they are perpendicular.

\[
\begin{align*}
\text{Line 1: } & y = mx + c_1 \\
\text{Line 2: } & y = nx + c_2
\end{align*}
\]
Mathematics Stage 5.2: Non-Linear Relationships

Name: ________________________________

Class: ________________________________

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

NUMBER AND ALGEBRA: NON-LINEAR RELATIONSHIPS

STUDENTS HAVE LEARNT TO

☐ Graph simple parabolas, exponentials and circles on the Cartesian plane using tables of values and digital technologies

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.2: NON-LINEAR RELATIONSHIPS

SYLLABUS REFERENCE

MA5.2-10NA - A student connects algebraic and graphical representations of simple non-linear relationships.

QUESTION 1

IDENTIFY, DRAW AND COMPARE GRAPHS OF PARABOLAS OF THE FORM Y = AX^2 + C

a) Graph and label the following sets of parabolas on the same Cartesian plane.

i) \( y = x^2 \) \( y = -x^2 \) \( y = 2x^2 \)

ii) \( y = x^2 \) \( y = x^2 + 2 \) \( y = x^2 - 1 \)
ii) Comment on any similarities or differences between the graphs including the co-efficient of the $x^2$ and the constant term.

---

b) Find the equation of a parabola with vertex at (0,-1).

---

DETERMINE THE EQUATION OF A PARABOLA

c) Find the x coordinate of the parabola $y = 2x^2 - 7$ whose y coordinate is 5.

---

DETERMINE THE EQUATION OF A PARABOLA

d) Determine the equation of the parabola below.
STAGE 5.2: NON-LINEAR RELATIONSHIPS

QUESTION 2

Sketch and describe the similarities and differences of the graphs.

a) \( y = 2^x \quad y = -2^x \quad y = 2^{-x} \)

b) \( y = 2^x \quad y = 2^x + 1 \quad y = 2^x - 1 \)
STAGE 5.2: NON-LINEAR RELATIONSHIPS

QUESTION 3  
RECOGNISE AND DESCRIBE EQUATIONS THAT REPRESENT CIRCLES

a) Find the centre and radius of a circle $x^2 + y^2 = 9$.

b) Write the equation of a circle with radius (0,0) and radius 2 units.

QUESTION 4  
MATCH GRAPHS OF STRAIGHT LINES, PARABOLAS, CIRCLES AND EXPONENTIALS TO THE APPROPRIATE EQUATIONS

a) Match the following graphs to their equation.

A

B

C

D

E

F

i) $y = 2^{-x}$

ii) $25 = x^2 + y^2$

iii) $y = -x^2 - 1$

iv) $y = x^2 + 3$

v) $x^2 + y^2 = 9$

vi) $y = x + 4$
STUDENTS HAVE LEARNT TO

- Solve problems involving areas of composite shapes by dissection into triangles, quadrilaterals, quadrants, semicircles and sectors
- Solve problems involving the surface area of rectangular and triangular prisms

SYLLABUS REFERENCE

MA5.2-11MG - A student calculates the surface areas of right prisms, cylinders and related composite solids.

QUESTION 1

CALCULATE THE SURFACE AREAS OF CYLINDERS

Calculate the surface area of the following:

a) (closed cylinder)

b) (closed half cylinder)
STAGE 5.2: AREA AND SURFACE AREA

c)  The following open cylindrical tin is empty and will be used to store stationery.

i) Calculate the surface area of the label around the tin.

ii) Calculate the surface area of the tin.

11 cm

Stationery

20 cm

78.6 mm (cylinder open at one end)

2.5 cm

QUESTION 2

The following open cylindrical tin is empty and will be used to store stationery.

a) Calculate the surface area of the label around the tin.

b) Calculate the surface area of the tin.
b) A swimming pool has the shape of a trapezoidal prism as shown in the diagram below. If this pool is to be tiled, find the cost of tiling the pool if tiles cost $45 per square metre.

![Diagram of a trapezoidal prism](image)

\[ \text{Area of pool} = \frac{1}{2} \times (16 + 50) \times 0.9 \times 50 \]

\[ \text{Cost} = \text{Area} \times \text{Cost per square metre} \]

\[ \text{Cost} = (35.75 \times 50) \times 45 \]

\[ \text{Cost} = 1608.75 \times 45 \]

\[ \text{Cost} = 72393.75 \]

\[ \text{Cost} = \$72393.75 \]

c) The curved roof of a green house is to be covered in shade cloth.

i) Calculate, correct to 1 decimal place, the area of shade cloth needed if there are no overlaps.

\[ \text{Area of roof} = \text{Length} \times \text{Width} \]

\[ \text{Area of roof} = 12 \times 4 \]

\[ \text{Area of roof} = 48 \]

\[ \text{Area of roof} = 48.0 \text{ square metres} \]

ii) Shade cloth is sold in 1.6 m wide rolls. How many linear metres of shade cloth are needed to cover the curved roof? Answer to the nearest 0.1 metres.

\[ \text{Width of roof} = 3 \text{ m} \]

\[ \frac{3}{1.6} \approx 1.875 \]

\[ \text{Answer} = 1.9 \text{ linear metres} \]
Mathematics Stage 5.2: Volume

Name: ____________________________

Class: ____________________________

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

MEASUREMENT AND GEOMETRY: VOLUME

STUDENTS HAVE LEARNT TO

☐ Visualise and draw different views of three dimensional objects
☐ Convert between metric units of volume and capacity
☐ Establish and use formulas to find volumes of right prisms and cylinders
☐ Solve problems involving volume and capacity

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.2: VOLUME

SYLLABUS REFERENCE

MA5.2-12MG - A student applies formulas to calculate the volume of composite solids composed of right prisms and cylinders.

QUESTION 1

FIND THE VOLUMES OF COMPOSITE SOLIDS

Calculate the volume area of the following:

a) b)
STAGE 5.2: VOLUME

The swimming pool below is 16 m long and 12 m wide. The depth of the water ranges from 1 m to 3 m.

Calculate the capacity of the pool in kilolitres.
QUESTION 3

SOLVE A VARIETY OF PRACTICAL PROBLEMS
RELATED TO THE VOLUMES AND CAPACITIES OF PRISMS

Rice crackers of diameter 38 mm are packed in a cardboard box of height 22 cm.
Calculate correct to 2 decimal places:

a) the volume of the crackers in the box.

b) the volume of the box.

c) the percentage of the box that is empty space.
Mathematics Stage 5.2: Right-Angled Triangles (Trigonometry)

Name: __________________________________________
Class: __________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

MEASUREMENT AND GEOMETRY: TRIGONOMETRY

STUDENTS HAVE LEARNT TO

☐ Use trigonometry to find sides and angles in right-angled triangles
  (angles measured to nearest degree)

☐ Apply trigonometry to solve problems involving right-angled triangles,
  including the use of angles of elevation and depression, from a given diagram

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

MA5.2-13MG - A student applies trigonometry to solve problems, including problems involving bearings.

STAGE 5.2: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

SYLLABUS REFERENCE

QUESTION 1 USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (INCLUDING ANGLES MEASURED TO NEAREST MINUTE)

Find the pronumeral in the following to 2 decimal places:

a) 

\[
\begin{align*}
\text{\angle A = 32^\circ 16' , x cm} \\
\text{25.6 cm}
\end{align*}
\]

b) 

\[
\begin{align*}
\text{\angle B = 20^\circ 32' , y cm} \\
\text{12 cm}
\end{align*}
\]
STAGE 5.2: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 2

USE TRIGONOMETRY TO FIND SIDES AND ANGLES IN RIGHT-ANGLED TRIANGLES (INCLUDING ANGLES MEASURED TO NEAREST MINUTE)

Find the following angles to the nearest minute:

a)  

\[ \theta \]

\[ \begin{array}{c}
22 \text{ cm} \\
15 \text{ cm}
\end{array} \]

b)  

\[ \theta \]

\[ \begin{array}{c}
15.5 \text{ cm} \\
23.5 \text{ cm}
\end{array} \]

QUESTION 3

APPLY TRIGONOMETRY TO SOLVE WORD PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES, INCLUDING THE USE OF THREE-Figure BEARINGS AND ANGLES OF ELEVATION AND DEPRESSION

a)  A 15 m ladder is leaning against a wall. The angle the ladder makes with the ground is 28°15'. How far is the foot of the ladder from the wall?

b)  The angle of depression from the top of a cliff is 58°. A boat out at sea is 125 m from the base of the cliff. Calculate the height of the cliff to the nearest metre.
c) The bearing of Q from P in the diagram below is S4°.

![Diagram]

Calculate the bearing of P from Q.

d) John starts walking from his camping site to collect wood. He walks 350 m north and then 230 m west and then he stops.

i) Draw a diagram of John’s path.

ii) How far is he away from camp to the nearest metre?

iii) Calculate his bearing from camp to the nearest minute.
Mathematics Stage 5.2: Properties of Geometric Figures

Name: 

Class: 

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

MEASUREMENT AND GEOMETRY: PROPERTIES OF GEOMETRIC FIGURES

STUDENTS HAVE LEARNT TO
- Identify similar figures and describe their properties informally
- Use scale factors and proportion statements to determine unknown lengths in similar figures
- Use scales on maps, plans and drawings

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

SYLLABUS REFERENCE

MA5.2-14MG - A student calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar.

QUESTION 1

WRITE FORMAL PROOFS OF THE CONGRUENCE OF TRIANGLES

a) State the four tests for congruent triangles.
b) Are the following triangles congruent? State the test used to justify that the triangles are congruent.

i) 

ii) 

c) In the diagram below:

i) prove that \(\triangle ABD \equiv \triangle CBD\)

ii) hence, prove that \(\angle DAB = \angle DCB\)
QUESTION 2

USE THE CONGRUENCE OF TRIANGLES TO PROVE PROPERTIES OF THE SPECIAL QUADRILATERALS

In the diagram below:

\[ \triangle PQS \equiv \triangle RSQ \]

i) prove that \( \triangle PQS \equiv \triangle RSQ \)

ii) hence, prove that PQRS is a parallelogram

QUESTION 3

DETERMINE THE MINIMUM CONDITIONS FOR TRIANGLES TO BE SIMILAR

List the tests to justify if two triangles are similar.

---

**Diagram:**

- Points Q, R, S, and P are connected to form a quadrilateral.

- \( \triangle PQS \equiv \triangle RSQ \)

- \( \triangle PQS \) and \( \triangle RSQ \) are congruent.

- PQRS is a parallelogram due to the congruence of triangles.
STAGE 5.2: PROPERTIES OF GEOMETRIC FIGURES

QUESTION 4

DETERMINE WHETHER TWO TRIANGLES ARE SIMILAR USING AN APPROPRIATE TEST

State the test that proves the triangles below are similar.

a)

b)

QUESTION 5

APPLY LOGICAL REASONING, INCLUDING THE USE OF SIMILARITY TO NUMERICAL EXERCISES INVOLVING PLANE SHAPES

Find the value of the pronumeral in the following:

---

---

---

---
QUESTION 6

A regular convex polygon has 20 sides, find:

i) the sum of its interior angles

ii) the size of each interior angle

iii) the size of each exterior angle
Mathematics Stage 5.2: Single Variable Data Analysis

Name: ________________________________
Class: ________________________________

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

STATISTICS AND PROBABILITY: SINGLE VARIABLE DATA ANALYSIS

STUDENTS HAVE LEARNT TO

☐ Construct and interpret back-to-back stem and leaf plots
☐ Describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi-modal’
☐ Compare two sets of numerical data in a display using mean, median and range
☐ Interpret and critically evaluate reports in the media and elsewhere that link claims to data displays and statistics

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

MA5.2-15SP - A student uses quartiles and box plots to compare sets of data, and evaluates sources of data.

QUESTION 1 DETERMINE QUARTILES AND INTERQUARTILE RANGE

For the following set of data:

{2 3 4 5 5 6 7 7 8 12}

a) determine the ‘five number summary’ (upper and lower extremes, median, upper and lower quartile).

b) determine the interquartile range for this set of data.
STAGE 5.2: SINGLE VARIABLE DATA ANALYSIS

CONSTRUCT, INTERPRET AND USE BOX PLOTS TO COMPARE SETS OF DATA

c) What percentage of data is contained within the interquartile range?

d) Construct a box plot for this data set?

e) What percentage of data is less than 4?

QUESTION 2

COMPARE SHAPES OF BOX PLOTS TO CORRESPONDING HISTOGRAMS AND DOT PLOTS

Draw a line and match the following histogram and dot plot to their corresponding box plots.

a)

b)

i)

ii)
QUESTION 3

DESCRIPT SIMILARITIES AND DIFFERENCES BETWEEN TWO SETS OF DATA DISPLAYED IN PARALLEL BOX PLOTS

Year 9 and year 12 students were surveyed on how much time they spent on revision per week. The data collected has been displayed in the parallel box plots below:

<table>
<thead>
<tr>
<th>Time Spent on revision per week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 9</td>
</tr>
<tr>
<td>Year 12</td>
</tr>
</tbody>
</table>

Number of hours

a) compare the range for year 9 and year 12.

b) calculate the interquartile range for both data sets.

c) find the median time spent on revision for the year 9 and year 12 students.

d) which class spent the longest time on revision? Comment on why this may be the case.

e) what percentage of students studied more than 7 hours in year 12?
Mathematics Stage 5.2: Bivariate Data Analysis

Name: ____________________________________________

Class: ____________________________________________

STAGE 5.2: BIVARIATE VARIABLE DATA ANALYSIS

SYLLABUS REFERENCE

MA5.2-16SP - A student investigates relationships between two statistical variables, including their relationship over time.

QUESTION 1

RECOGNISE THE DIFFERENCE BETWEEN AN INDEPENDENT VARIABLE AND ITS DEPENDENT VARIABLE

Consider the following statements and identify the dependent variable and the independent variable in each:

a) The amount of rainfall varies with each month of the year.

Dependent variable: _____________________________

Independent variable: ____________________________

b) The heights and hand-spans of a group of students.

Dependent variable: _____________________________

Independent variable: ____________________________

QUESTION 2

RECOGNISE THE DIFFERENCE BETWEEN AN INDEPENDENT VARIABLE AND ITS DEPENDENT VARIABLE

Which type of graph is the most appropriate method of representing data collected over time, and why?

__________________________________________________

__________________________________________________
STAGE 5.2: BIVARIATE VARIABLE DATA ANALYSIS

QUESTION 3

RECOGNISE THE DIFFERENCE BETWEEN AN INDEPENDENT VARIABLE AND ITS DEPENDENT VARIABLE

The following table shows the estimated Australian rainfall for 2014 by month.

<table>
<thead>
<tr>
<th>MONTH</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAINFALL (MM)</td>
<td>115</td>
<td>41</td>
<td>45</td>
<td>28</td>
<td>22</td>
<td>20</td>
<td>17</td>
<td>18</td>
<td>5</td>
<td>10</td>
<td>37</td>
<td>82</td>
</tr>
</tbody>
</table>

a) Construct a line graph for the data above using time as the independent variable.

b) Which month was the driest?

c) Which month was the wettest?

d) Analyse and comment on any trends expressed in the graph.

 QUESTION 4

DESCRIBE, INFORMALLY, THE STRENGTH AND DIRECTION OF THE RELATIONSHIP BETWEEN TWO VARIABLES DISPLAYED IN A SCATTER PLOT

Classify the following as either having strong or weak, positive or negative or no correlation.
STAGE 5.2: BIVARIATE VARIABLE DATA ANALYSIS

QUESTION 5

INTERPRET SCATTER PLOTS OF TWO NUMERICAL VARIABLES

The scatter plot below shows the final exam results versus topic tests results.

a) Describe the relationship between the two variables.

b) Estimate the final exam result of a student who achieved 5 in their topic tests.

c) Analyse and comment on any trends expressed in the scatter plot.
STAGE 5.2: BIVARIATE VARIABLE DATA ANALYSIS

QUESTION 6

DESCRIBE, INFORMALLY, THE STRENGTH AND DIRECTION OF THE RELATIONSHIP BETWEEN TWO VARIABLES DISPLAYED IN A SCATTER PLOT

John took a long journey home and recorded the distance travelled as well as the remaining fuel in the tank. The following table shows the results.

<table>
<thead>
<tr>
<th>DISTANCE TRAVELLED (KM)</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUEL IN TANK (LITRES)</td>
<td>100</td>
<td>92</td>
<td>84</td>
<td>76</td>
<td>68</td>
<td>58</td>
<td>50</td>
<td>40</td>
<td>32</td>
</tr>
</tbody>
</table>

a) Construct a scatterplot for the data above using the graph below.

b) Comment on the relationship between the two variables.
Mathematics Stage 5.2: Probability

Name: ____________________________________________

Class: ____________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.1 KEY IDEAS

STATISTICS AND PROBABILITY: PROBABILITY

STUDENTS HAVE LEARNT TO

☐ Calculate relative frequencies to estimate probabilities
☐ Calculate probabilities from Venn diagrams and two-way tables

SYLLABUS REFERENCE

MA5.2-17SP - A student describes and calculates probabilities in multistep chance experiments.

STAGE 5.2: PROBABILITY

QUESTION 1

a) A die is rolled and a coin is thrown. List the sample space.

b) What is the probability of obtaining a head and a 6?
STAGE 5.2: PROBABILITY

QUESTION 2
CALCULATE PROBABILITIES FOR SIMPLE AND COMPOUND EVENTS

A bag contains 4 blue counters and 3 red counters. A counter is picked out of the bag and then returned before a second counter is chosen.

a) What is the probability of choosing a blue counter on the first selection?

b) What is the probability of choosing a blue counter on both selections?

c) Calculate the probability of choosing a blue and then a red counter.

QUESTION 3
RECORD RESULTS OF TWO- AND THREE-STEP CHANCE EXPERIMENTS, AND DETERMINE PROBABILITIES OF EVENTS

A coin is tossed three times.

a) Complete the tree diagram below and list the sample space.

b) What is the probability of obtaining three heads?

c) What is the probability of obtaining at least one tail?
QUESTION 4

Jessica has a bag of jelly beans made up of 2 green, 3 red and 4 orange jelly beans, she takes a jelly bean from the bag and eats it.

a) Calculate the probability that Jessica eats a green jelly bean.

b) Calculate the probability that Jessica eats a red jelly bean next.

c) If a green and red jellybean have been eaten from the bag, calculate the probability of obtaining an orange jellybean on the third attempt.

QUESTION 5

Distinguish whether the following pairs of events are dependent or independent:

a) rolling a die to obtain a 4 and rolling a second die to obtain an even number.

b) tossing a coin to obtain a head and then tossing a coin to obtain a tail.

c) a person who is short sighted and a person wearing glasses.
QUESTION 6

Calculate probabilities of events where a condition is given that restricts the sample space

Given that a number greater than 2 has been rolled on a fair six sided die:

a) list the sample space.

b) calculate the probability that this number was 6.

c) calculate the probability that this number was 1.

QUESTION 7

Critically evaluate conditional statements in chance situations

a) Explain why if you toss a 1 on a six sided die, the probability of obtaining a 1 on the second toss remains the same.

b) Explain why the statement ‘If you obtain a head on three consecutive tosses of a coin, then there is a greater chance of obtaining a tail on the next toss’ is incorrect.
Mathematics Stage 5.3: Rates and Ratios

Name: ____________________________

Class: ____________________________

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

NUMBER AND ALGEBRA: RATES AND RATIOS

STUDENTS HAVE LEARNT TO

☐ Solve problems involving constant rates of change
☐ Interpret graphs of change where the relationship between the variables is constant
☐ Construct graphs of direct linear proportion

SYLLABUS REFERENCE

MA5.3-4NA - A student draws, interprets and analyses graphs of physical phenomena.

STAGE 5.3: RATES AND RATIOS

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

QUESTION 1

INTERPRET AND DESCRIBE GRAPHS OF CHANGE WHERE THE RELATIONSHIP BETWEEN THE VARIABLES IS NOT CONSTANT

a) A truck driver is driving at a certain speed and then he decreases his speed, which graph represents this?

A

Distance

Time

B

Distance

Time

C

Distance

Time
STAGE 5.3: RATES AND RATIOS

QUESTION 1

b) A cylindrical tank is filled with water. Which graph represents this?

A)

B)

C)

QUESTION 2

A cyclist left home early and went for a ride. The travel graph below shows his journey:

a) what time did his journey finish?

b) how long did his journey take?

c) what time did he rest? How long was the rest for?

d) between what times was he travelling the fastest?

e) what was the average speed of the cyclist during his journey?
STAGE 5.3: RATES AND RATIOS

QUESTION 3

SKETCH GRAPHS OF PHYSICAL PHENOMENA

a) A ball is dropped from the top of a 10 metre building to the ground and then it eventually stops. Draw a graph that represents the height of the ball above the ground over time.

b) Water is being poured into the flask below at a constant rate. Sketch a graph that represents the level of the water in the flask over time.
QUESTION 4

INTERPRET GRAPHS, MAKING SENSIBLE STATEMENTS
ABOUT THE RATE OF INCREASE OR DECREASE

Jane kicks a football high up in the air and it lands on the neighbour’s shed.

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Time (in seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

a) How high was the shed?

b) How long did it take for the ball to land on the shed?

c) What was the maximum height reached by the ball?

d) When was the height decreasing?

e) What was the average speed of the ball from when it was kicked in the air until it landed on the shed?
**STAGE 5.3: RATES AND RATIOS**

**QUESTION 5**

Interpret graphs, making sensible statements about the rate of increase or decrease.

Describe what is happening in the graph below.

**A Trucks Journey**

![Graph](image)

**Time**

**Fuel**

**QUESTION 6**

Describe qualitatively the rate of change of a graph.

Match the graphs to their corresponding descriptions:

- Increasing at a constant rate
- Decreasing at a decreasing rate
- Decreasing at a constant rate
- Increasing at an increasing rate
- Increasing at a decreasing rate
- Decreasing at an increasing rate
Mathematics Stage 5.3: Algebraic Techniques

Name: 

Class: 

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

NUMBER AND ALGEBRA: ALGEBRAIC TECHNIQUES

STUDENTS HAVE LEARNT TO

☐ Apply the four operations to simplify simple algebraic fractions
☐ Expand and simplify algebraic expressions where appropriate, including binomial products
☐ Factorisemonic quadratic trinomial expressions

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: ALGEBRAIC TECHNIQUES

SYLLABUS REFERENCE

MA5.3-5NA - A student selects and applies appropriate algebraic techniques to operate with algebraic expressions.

QUESTION 1

ADD AND SUBTRACT ALGEBRAIC FRACTIONS WITH NUMERICAL DENOMINATORS, INCLUDING THOSE WITH BINOMIAL NUMERATORS

Simplify the following:

\[ \frac{3x + 2}{4} + \frac{x - 7}{2} \]

\[ \frac{2x}{3} - \frac{x + 3}{4} \]
STAGE 5.3: RATES AND RATIOS

QUESTION 2

EXPAND BINOMIAL PRODUCTS USING A VARIETY OF STRATEGIES

Expand and simplify the following:

a) \((y + 7)^2\)

b) \((3y – 4)^2\)

c) \((2x – 1)(2x + 1)\)

d) \((3y + 2)(4 – y) + 4y + 5\)

e) \((2x – y)^2 – (2x + y)^2\)
QUESTION 3

Factorise the following:

a) $4y^2 + 12y$  

b) $y(x + 3) + 8(x + 3)$

c) $2xy + 8x + 4y + 16$  
d) $a^2 - 16$

e) $4y^2 - 25x^2$  
f) $3d^3 - 3d$

g) $y^2 + 6y + 9$  
h) $x^2 + 9x - 36$

i) $6y^2 + 20y + 14$  
j) $4x^2 - 20x + 25$
STAGE 5.3: RATES AND RATIOS

QUESTION 4

Simplify:

a) \( \frac{x^2 + 3x + 2}{x + 2} \)

b) \( \frac{3m - 6}{4} \times \frac{8m}{m^2 - 2m} \)

c) \( \frac{4}{x^2 - 9} \div \frac{2}{3x + 9} \)

d) \( \frac{1}{y - 2} - \frac{2}{y} \)

e) \( \frac{4}{x^2 + x} - \frac{3}{x^2 - 1} \)
BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

NUMBER AND ALGEBRA: INDICES

STUDENTS HAVE LEARNT TO

☐ Convert algebraic expressions with negative indices to expressions with positive indices and vice versa

☐ Simplify algebraic expressions involving positive, negative and zero indices

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: INDICES

SYLLABUS REFERENCE

MA5.3-6NA - A student performs operations with surds and indices.

QUESTION 1

Define a rational number:
QUESTION 2

Distinguish between rational and irrational numbers.

Determine whether the following numbers are rational or irrational:

a) 0.32

b) $2\frac{2}{3}$

c) $\frac{\pi}{2}$

d) $\sqrt[3]{34}$

e) 0.7

QUESTION 3

Write the following recurring decimals in fraction form.

Write the following recurring decimals as fractions:

a) 0.2

b) 0.43

c) 1.352
QUESTION 4

PERFORM OPERATIONS WITH SURDS

Simplify the following surds:

a) \((\sqrt{8})^2\)

b) \((-4\sqrt{6})^2\)

c) \(\sqrt{\frac{81}{64}}\)

d) \(\sqrt{2} \times \sqrt{3}\)

e) \(\sqrt{8}\)

f) \(6\sqrt{20}\)

g) \(2\sqrt{3} + 3\sqrt{12}\)

h) \(2\sqrt{90} + \sqrt{15}\)

i) \(\frac{4\sqrt{12} \times 3\sqrt{6}}{2\sqrt{27}}\)

QUESTION 5

SIMPLIFY EXPRESSIONS INVOLVING SURDS

Simplify the following:

a) \(6\sqrt{5} + 2\sqrt{5}\)

b) \(-5\sqrt{7} + 5\sqrt{2} - 3\sqrt{7}\)

c) \(4\sqrt{8} + 2\sqrt{32}\)

d) \(\sqrt{2} (\sqrt{4} + \sqrt{7})\)

e) \((2 - 3\sqrt{10})(\sqrt{5} - 4\sqrt{2})\)

f) \((3\sqrt{7} - \sqrt{5})^2\)

g) \((\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})\)

h) \((2\sqrt{5} + 4)(2\sqrt{5} - 4)\)
QUESTION 6

Explain why: $\sqrt{3} + \sqrt{4} \neq \sqrt{7}$

---

QUESTION 7

RATIONALISE THE DENOMINATORS OF SURDS

Rationalise the denominator for the following (simplifying answers where necessary):

a) $\frac{1}{\sqrt{5}}$  

b) $\frac{8}{2\sqrt{10}}$  

c) $\frac{2\sqrt{2} - \sqrt{3}}{\sqrt{6}}$

---

QUESTION 8

Determine the perpendicular height for the triangle below in exact surd form:

---
STAGE 5.3: INDICES

QUESTION 9
Write each of the following with a fractional index:

a) $\sqrt[5]{y}$

b) $\sqrt[3]{64w^9}$

c) $\frac{1}{\sqrt[6]{81x^7}}$

QUESTION 10
Write each of the following in surd form:

a) $x^\frac{5}{7}$

b) $4y^\frac{3}{4}$

c) $(8m)^{\frac{3}{4}}$

d) $6y^{-\frac{3}{5}}$

QUESTION 11
Evaluate the following (answering in exact form):

a) $81^\frac{1}{4}$

b) $125^{-\frac{2}{3}}$

c) $16^{-\frac{3}{2}}$
STAGE 5.3: INDICES

QUESTION 12

Use a calculator to evaluate the following correct to 3 decimal places:

a) \(20^{\frac{2}{3}}\)  

b) \(9^{\frac{1}{3}}\)  

c) \(\left(\frac{9}{64}\right)^{\frac{1}{3}}\)
# Mathematics Stage 5.3: Equations

**Background Knowledge: Stage 5.2 Key Ideas**

**Number and Algebra: Equations**

**Students Have Learnt To**

- Solve linear equations involving grouping symbols
- Solve linear equations involving algebraic fractions
- Solve quadratic equations of the form $ax^2 = c$
- Solve quadratic equations of the form $ax^2 + bx + c = 0$ (where $a = 1$) using factors
- Solve equations resulting from substitution into formulas
- Solve word problems using linear equations
- Solve linear inequalities
- Solve linear simultaneous equations using algebraic and graphical techniques

**Stage 5.3: Equations**

**Syllabus Reference**

**MA5.3-7NA** - A student solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations.

**Question 1**

Solve the following:

<table>
<thead>
<tr>
<th>$a)$ $\frac{3x - 6}{5} - \frac{x + 7}{3} = 2$</th>
<th>$b)$ $\frac{y - 3}{3} - \frac{4y + 3}{2} = \frac{1}{4}$</th>
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</tbody>
</table>
STAGE 5.3: EQUATIONS

QUESTION 2

SOLVE QUADRATIC EQUATIONS BY FACTORISING

Solve the following:

a) \( x^2 - 16 = 0 \)

b) \( 3x^2 = 4 \)

c) \( 4x^2 - 8x = 0 \)

d) \( y^2 + 8y + 7 = 0 \)

e) \( x(x - 4) = -4 \)

f) \( (y - 2)^2 = 9 \)
### STAGE 5.3: EQUATIONS

#### QUESTION 3
Solve the following by completing the square:

a) \( p^2 - 6p + 2 = 0 \)

#### QUESTION 4
Use the quadratic formula to solve the following: (leave answers in surd form)

a) \( x^2 - 8x - 4 = 0 \)

b) \( 3y^2 - 10y + 4 = 0 \)

#### QUESTION 5
Solve the following by substituting \( u = x^2 \).

a) \( x^4 - 13x^2 + 36 = 0 \)

b) \( 4x^4 - 12x^2 - 16 = 0 \)
STAGE 5.3: EQUATIONS

QUESTION 6
SOLVE SIMPLE CUBIC EQUATIONS OF THE FORM $AX^3 = K$

Solve the following:

a) $x^3 = -8000$

b) $8x^3 = 216$

c) $11x^3 = 31$

QUESTION 7
REARRANGE LITERAL EQUATIONS

Change the subject of the formulas below to the pronumerals indicated in the brackets:

a) $v = u + at$  \hspace{1cm} \text{(t)}

b) $F = \frac{9C}{5} + 32$  \hspace{1cm} \text{(C)}

c) $x = \sqrt{b^2 - 4ac}$  \hspace{1cm} \text{(b)}
STAGE 5.3: EQUATIONS

QUESTION 8

SOLVE SIMULTANEOUS EQUATIONS WHERE ONE EQUATION IS NON-LINEAR

Solve the following pairs of simultaneous equations:

a) \[
\begin{align*}
  y &= x^2 - x - 2 \\
  y &= x + 6
\end{align*}
\]

b) \[
\begin{align*}
  y &= x + 2 \\
  y &= \frac{2}{x}
\end{align*}
\]
Mathematics Stage 5.3: Linear Relationships

Name: ____________________________________________

Class: ____________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

NUMBER AND ALGEBRA: LINEAR RELATIONSHIPS

STUDENTS HAVE LEARNT TO

☐ Apply the gradient-intercept form of the equation of a straight line to interpret and graph straight lines

☐ Apply the properties of the gradients of parallel and perpendicular lines on the Cartesian plane

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: LINEAR RELATIONSHIPS

SYLLABUS REFERENCE

MA5.3-8NA - A student uses formulas to find midpoint, gradient and distance on the Cartesian plane, and applies standard forms of the equation of a straight line.

QUESTION 1

USE FORMULAS TO FIND THE MIDPOINT, GRADIENT AND LENGTH OF INTERVALS ON THE CARTESIAN PLANE

A(1,5)

B(-4,-1)
STAGE 5.3: LINEAR RELATIONSHIPS

QUESTION 2

USE FORMULAS TO FIND THE MIDPOINT, GRADIENT AND LENGTH OF INTERVALS ON THE CARTESIAN PLANE

For the interval AB find the following:

a) the midpoint.  
b) the gradient.

c) the length.

QUESTION 3

APPLY VARIOUS STANDARD FORMS OF THE EQUATION OF A STRAIGHT LINE

a) Find the equation of the line that has a gradient of -2 and passes through the point (2,3). Leave answer in gradient intercept form.

b) Find the equation of the line that passes through the points A(-3,-1) and B(2,5). Leave answer in general form.
STAGE 5.3: LINEAR RELATIONSHIPS

QUESTION 4

SOLVE PROBLEMS INVOLVING PARALLEL AND PERPENDICULAR LINES

a) Find the equation of the line that is parallel to the line $y = 2x + 1$ and passes through the point (3,2).

b) Determine whether the line $y = 3x + 1$ is perpendicular to the line $x + 3y - 6 = 0$.

QUESTION 5

SOLVE A VARIETY OF PROBLEMS BY APPLYING COORDINATE GEOMETRY FORMULAS

a) Show that the points A (-1,4), B(3,1) and C(2,8) form a right-angled triangle.

b) Show that the points P(-7,2), Q(2,-7), R(5,-4) and S(-4,5) form a rectangle.
Mathematics Stage 5.3: Non-Linear Relationships

Name: __________________________________________________________

Class: __________________________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

NUMBER AND ALGEBRA: NON-LINEAR RELATIONSHIPS

STUDENTS HAVE LEARNT TO

☐ Identify, draw and compare graphs of parabolas of the form \( y = ax^2 + c \)

☐ Identify, graph and compare exponential curves and circles

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME
STAGE 5.3: NON-LINEAR RELATIONSHIPS

SYLLABUS REFERENCE
MA5.3-9NA - A student sketches and interprets a variety of non-linear relationships.

QUESTION 1

For the parabola \( y = x^2 - 2x - 3 \):

a) find the x and y intercepts.

b) find the equation of the axis of symmetry.

c) determine the minimum value.

d) state the coordinates of the vertex.

e) Graph the parabola on a Cartesian plane.
STAGE 5.3: NON-LINEAR RELATIONSHIPS

QUESTION 2

Sketch and describe the similarities and differences of the graphs and what causes those differences:

a) \( y = \frac{1}{x} \) \( y = \frac{-1}{x} \) \( y = \frac{3}{x} \)

b) \( y = \frac{1}{x} \) \( y = \frac{1}{x} + 1 \) \( y = \frac{1}{x-2} \)
STAGE 5.3: NON-LINEAR RELATIONSHIPS

QUESTION 3

State the asymptotes of the following hyperbolas:

a) \( y = \frac{1}{x} + 4 \)  

b) \( y = \frac{1}{x + 3} \)

QUESTION 4

DESCRIBE, INTERPRET AND SKETCH CUBIC FUNCTIONS

Graph and compare the features of the following functions:

\( y = x^3 \)  \( y = 2x^3 \)  \( y = x^3 + 2 \)  \( y = (x - 4)(x - 2)(x + 1) \)
STAGE 5.3: NON-LINEAR RELATIONSHIPS

QUESTION 5

RECOGNISE AND DESCRIBE EQUATIONS THAT REPRESENT CIRCLES WITH CENTRE \((h,k)\) AND RADIUS \(r\)

a) Determine the centre and radius of the circle \((x - 2)^2 + (y + 3)^2 = 16\).

b) Find the centre and radius of the circle: \(x^2 + 2x + y^2 - 2y + 1 = 0\) by completing the square.
STAGE 5.3: NON-LINEAR RELATIONSHIPS

QUESTION 6

GRAPH CURVES OF THE FORM $Y = AX^n + K$

a) Graph the following on a separate number plane by first determining the x and y intercepts.

i) $y = x^5 + 3$

ii) $y = x^4 + 1$

b) What do you notice about the odd and even powers of these functions and their graphs?
Determine the point of intersection of the parabola $y = x^2 + 1$ and the line $y = -2x + 4$ both graphically and algebraically.
QUESTION 8

Determine the points of intersection of a line with a parabola.

Determine a possible equation for each given graph:

a) 

b) 

c) 

d)
Mathematics Stage 5.3: Polynomials

Name: ______________________________

Class: ______________________________

BACKGROUND KNOWLEDGE: STAGE 5.3 KEY IDEAS

NUMBER AND ALGEBRA: POLYNOMIALS

STUDENTS HAVE LEARNT TO

☐ Draw, interpret and compare graphs of parabolas of the form \( y = ax^2 + bx + c \) using a variety of techniques.

☐ Determine the equation of the axis of symmetry and the coordinates of the vertex of a parabola.

☐ Draw, interpret and compare graphs of hyperbolas, exponentials, circles and simple cubic functions.

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

MA5.3-10NA - A student recognizes, describes and sketches polynomials, and applies the factor and remainder theorems to solve problems.

STAGE 5.3: POLYNOMIALS

SYLLABUS REFERENCE

QUESTION 1


Determine the following for the polynomial \( y = 2x^5 - 3x^3 + 5x + 9 \):

a) the leading term. ____________________________________________________________

b) the degree. _________________________________________________________________

c) the leading coefficient. _____________________________________________________

d) is it monic. _______________________________________________________________

e) the constant term. __________________________________________________________
STAGE 5.3: POLYNOMIALS

QUESTION 2  ADD, SUBTRACT, MULTIPLY AND DIVIDE POLYNOMIALS

Evaluate the following for the polynomials $P(x) = 3x^3 + 2x^2 + 5$ and $Q(x) = x^3 - 2$:

a) $P(x) + Q(x)$ 

b) $P(x) - Q(x)$

QUESTION 3  ADD, SUBTRACT, MULTIPLY AND DIVIDE POLYNOMIALS

Evaluate the following:

$(2x - 4) \times (5x^2y + 4xy - x)$

QUESTION 4  DIVIDE POLYNOMIALS BY LINEAR EXPRESSIONS

Divide $P(x) = x^3 - 2x + 4$ by $A(x) = (x - 2)$. Then express $P(x)$ in the form $P(x) = A(x) \cdot Q(x) + R(x)$. 
STAGE 5.3: POLYNOMIALS

QUESTION 5  APPLY THE FACTOR THEOREM

Find the remainder when:

a) \((2y^2 + 7y - 2) \div (y + 7)\)

b) \((m^5 + 5m^3 - 1) \div (m - 3)\)

QUESTION 6  APPLY THE REMAINDER THEOREM

Show that \(p - 2\) is a factor of \(p^3 - 4p^2 + p + 6\).
QUESTION 7

Find the zeros of the following polynomials and sketch the graph of each:

a) \( y = (x + 2)(x - 3)(x - 4) \)

b) \( y = x^3 + 2x^2 + x \)

c) \( y = 2x^3 + 3x^2 - 12x - 13 \)
Mathematics Stage 5.3: Logarithms

STUDENTS HAVE LEARNT TO

- Draw, interpret and compare graphs of parabolas of the form \( y = ax^2 + bx + c \) using a variety of techniques
- Determine the equation of the axis of symmetry and the coordinates of the vertex of a parabola
- Draw, interpret and compare graphs of hyperbolas, exponentials, circles and simple cubic functions

SYLLABUS REFERENCE

MA5.3-11NA - A student uses the definition of a logarithm to establish and apply the laws of logarithms.

QUESTION 1

Translate the following into logarithmic expressions:

a) \( 16 = 4^2 \)  

b) \( \frac{1}{3} = 3^{-1} \)  

c) \( 4^7 = 8 \)
STAGE 5.3: LOGARITHMS

QUESTION 2

Apply the law of logarithms to simplify the following expressions:

a) \( \log_3{27} \)

b) \( \log_{25}{5} \)

c) \( \log_4{8} + \log_4{2} \)

d) \( \log_{25}{50} + 3\log_{25}{2} \)

e) \( \log_4{16} - 2\log_4{2} \)

f) \( 4\log_a{a} - \log_a{a^3} \)

QUESTION 3

Solve the following:

a) \( 4^t = 64 \)

b) \( 3^{t+1} = 9 \)

c) \( \log_{10}{1000} = x \)

d) \( \log_3{x} = 5 \)
STAGE 5.3: LOGARITHMS

QUESTION 4

Graph and compare the graphs of the inverse functions $y = A^x$ and $y = \log_A x$

Graph and compare the features of the following functions:

a) $y = 2^x$ and $y = \log_2 x$

b) $y = 3^x$ and $y = \log_3 x$
Mathematics Stage 5.3: Functions and Other Graphs

Name: _______________________

Class: _______________________

BACKGROUND KNOWLEDGE: STAGE 5.3 KEY IDEAS

NUMBER AND ALGEBRA: FUNCTIONS AND OTHER GRAPHS

STUDENTS HAVE LEARNT TO

☐ Draw, interpret and compare graphs of parabolas of the form $y = ax^2 + bx + c$ using a variety of techniques

☐ Determine the equation of the axis of symmetry and the coordinates of the vertex of a parabola

☐ Draw, interpret and compare graphs of hyperbolas, exponentials, circles and simple cubic functions

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: FUNCTIONS AND OTHER GRAPHS

SYLLABUS REFERENCE

MA5.3-12NA - A student uses function notation to describe and sketch functions.

QUESTION 1

DETERMINE WHETHER A GRAPH REPRESENTS A FUNCTION

Determine whether the following graphs are functions:

a)  

b)
STAGE 5.3: FUNCTIONS AND OTHER GRAPHS

QUESTION 2  USE FUNCTION NOTATION

a) If \( f(x) = y^2 + 2y + 1 \), find \( f(-2) \) and \( f(1) \).

b) If \( f(x) = 2x + 3 \), find \( x \) when \( f(x) = 15 \).
STAGE 5.3: FUNCTIONS AND OTHER GRAPHS

QUESTION 3

Determine whether the following have inverse functions; if so find its inverse:

a) \( f(x) = 2x - 3 \)

b) \( f(x) = -\frac{1}{3}x + 2 \)

c) \( f(x) = 3x^2 - 4 \)

d) \( f(x) = \frac{-x - 2}{x + 4} \)


QUESTION 4

Find the inverse \( f^{-1}(x) \) of the function \( f(x) = x + 1 \) and hence sketch both functions on the same Cartesian plane. What do you notice?
STAGE 5.3: FUNCTIONS AND OTHER GRAPHS

QUESTION 5

SKETCH THE GRAPHS OF $y = f(x) + k$ AND $y = f(x - a)$, GIVEN THE GRAPH OF $y = f(x)$

If $f(x) = x^2$, then graph:

a) $y = f(x)$

b) $y = f(x) + 2$

c) $y = f(x - 4)$
QUESTION 1 SURFACE AREA OF RIGHT PYRAMIDS

A square pyramid is pictured below:

a) determine the perpendicular height of EF in ΔEAB.

b) calculate the surface area of the pyramid.
STAGE 5.3: AREA AND SURFACE AREA

QUESTION 2

a) Calculate the curved surface area of the cone.

b) Find the total surface area of the cone.
STAGE 5.3: AREA AND SURFACE AREA

QUESTION 3

SURFACE AREA OF A SPHERE

a) Calculate the surface area of the following:

i) 20.5 cm

ii)

b) The surface area of a sphere is 45 cm$^2$. Calculate the radius of the sphere.


STAGE 5.3: AREA AND SURFACE AREA

QUESTION 4

SURFACE AREA OF COMPOSITE SOLIDS

Calculate the surface area of the following composite shapes:

a)

b)

20.4 cm

35.6 cm

5 cm

6 cm

4 cm

4 cm
A wedding cake has three tiers as indicated in the diagram below. Each tier is 10 cm high. The layers have radii 22 cm, 17 cm and 12 cm respectively. Find the total surface area of the cake, correct to the nearest cm$^2$. 

STAGE 5.3: AREA AND SURFACE AREA

QUESTION 5

SOLVE PRACTICAL PROBLEMS INVOLVING THE SURFACE AREAS OF SOLIDS

A wedding cake has three tiers as indicated in the diagram below. Each tier is 10 cm high. The layers have radii 22 cm, 17 cm and 12 cm respectively. Find the total surface area of the cake, correct to the nearest cm$^2$. 

[Diagram of a wedding cake with three tiers labeled 22 cm, 17 cm, and 12 cm in radius, and 10 cm in height.]

---
Mathematics Stage 5.3: Volume

Name: __________________________________________________________

Class: __________________________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

MEASUREMENT AND GEOMETRY: VOLUME
STUDENTS HAVE LEARNT TO

☐ Solve problems involving volume and capacity of prisms, cylinders and related composite solids

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: VOLUME

SYLLABUS REFERENCE

MA5.3-14MG - A student applies formulas to find the volumes of right pyramids, right cones, spheres and related composite solids.

QUESTION 1

SOLVE PROBLEMS INVOLVING VOLUME OF RIGHT PYRAMIDS, RIGHT CONES, SPHERES AND RELATED COMPOSITE SOLIDS

Calculate the volume of the following:

a)
STAGE 5.3: VOLUME

b)  

\[ \text{Volume of a sphere} \]

\[ \frac{4}{3} \pi r^3 \]

\[ \frac{4}{3} \pi (12.75)^3 \]

\[ 2358.8 \text{ cm}^3 \]

c)  

\[ \text{Volume of a cone} \]

\[ \frac{1}{3} \pi r^2 h \]

\[ \frac{1}{3} \pi (15.4)^2 (25.6) \]

\[ 956.4 \text{ cm}^3 \]

d)  

\[ \text{Volume of a cylinder} \]

\[ \pi r^2 h \]

\[ \pi (34)^2 (23.5) \]

\[ 2735.8 \text{ cm}^3 \]
STAGE 5.3: VOLUME

QUESTION 2

APPLYING VOLUME FORMULAS

Find the radius of the cone whose volume is 1365 cm$^3$ and has a perpendicular height of 35 cm.

QUESTION 3

VOLUME AND CAPACITY OF COMPOSITE SOLIDS

Find the volume of the following and the capacity that it holds to the nearest litre:

- 34 cm
- 64 cm
A rectangular prism is to fit as many spherical balls as possible:

a) How many spherical balls can it fit?

b) What percentage of the box will be empty?
Mathematics Stage 5.3: Trigonometry

Name: ____________________________

Class: ____________________________

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

MEASUREMENT AND GEOMETRY: TRIGONOMETRY

STUDENTS HAVE LEARNT TO

☐ Use trigonometry to find sides and angles in right-angled triangles (including angles measured to nearest minute or second)

☐ Apply trigonometry to solve word problems involving right-angled triangles, including the use of three-figure bearings and angles of elevation and depression

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

SYLLABUS REFERENCE

MA5.3-15MG - A student applies trigonometry to solve problems, including problems involving bearings.

QUESTION 1

SOLVE THREE-DIMENSIONAL PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES

A rectangular prism has dimensions 28 cm, 40 cm and 75 cm. An umbrella will be packaged in the box below:

A 75 cm  

40 cm  

28 cm

θ

a) What is the longest length that the umbrella can be?

b) Calculate the angle θ that BF makes with BE in the plane BDFH.
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 2

SOLVE THREE-DIMENSIONAL PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES

The pyramid below has a square base ABCD with an area of 16 cm². ED, EB, EA and EC have a side length of 20 cm. EF is the perpendicular height of the pyramid. Calculate EF in exact form.
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 3

From a point B, due south of the flagpole 150 m tall on level ground, the angle of elevation of the top of the flagpole is 45°. The top of the same flagpole is observed with an angle of elevation of 32° from a point C, due east of the flagpole.

a) Sketch a diagram to portray the above information.

b) Calculate the distance from B to C.
QUESTION 4

SKETCH TRIGONOMETRIC FUNCTIONS

Sketch the following trigonometric graphs:

a) \( y = \sin x \) for \( 0^\circ \leq x \leq 360^\circ \)

b) \( y = \cos x \) for \( -180^\circ \leq x \leq 180^\circ \)

c) \( \tan x \) for \( 0^\circ \leq x \leq 360^\circ \)
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 5

Evaluate the exact values of the following:

a) \( \sin 60^\circ + \cos 30^\circ \)

b) \( \cos^2 60^\circ + \sin^2 30^\circ \)

c) \( \sin 120^\circ \)

d) \( \sin 225^\circ \)

e) \( \frac{\sin 45^\circ}{\cos 60^\circ} \)

QUESTION 6

Evaluate the exact length of \( y \).

<table>
<thead>
<tr>
<th>60°</th>
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</thead>
<tbody>
<tr>
<td>11</td>
</tr>
<tr>
<td>y</td>
</tr>
</tbody>
</table>
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 7

SOLVE PROBLEMS IN RIGHT-ANGLED TRIANGLES USING EXACT VALUES

a) A boat ramp is to be made with an angle of 45° and base length 8 m. What is the exact length of the surface of the ramp?

b) Find the exact length of AC.
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 8

SOLVE SIMPLE TRIGONOMETRIC EQUATIONS

Solve the following for $0^\circ \leq \theta \leq 360^\circ$:

\[
a) \sin \theta = \frac{1}{\sqrt{2}} \\
b) 2 \cos \theta = \sqrt{3}
\]

---

QUESTION 9

ANGLE OF INCLINATION

Find the angle of inclination that a line with gradient 0.8 makes with the x-axis in a positive direction to the nearest degree.
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 10

Find the value of the pronumeral in the following diagrams:

a) (correct to 1 decimal place)

b) (correct to the nearest minute)

c) (correct to 3 significant figure)
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

d) (correct to the nearest degree)

![Diagram of a right-angled triangle with sides 10 cm, 6 cm, and an angle of 33°.]

- a) Find the length of BD.
- b) Determine the size of θ to the nearest degree.

---

**QUESTION 11**

PROBLEMS INVOLVING MORE THAN ONE TRIANGLE

![Diagram of a complex figure involving multiple triangles.]

- a) Find the length of BD.
- b) Determine the size of θ to the nearest degree.
STAGE 5.3: TRIGONOMETRY - RIGHT-ANGLED TRIANGLES

QUESTION 12

SOLVE PRACTICAL PROBLEMS THAT INVOLVE NON-RIGHT-ANGLED TRIANGLES

a) A parallelogram has sides 11 cm and 5 cm, and one interior angle 79°25'. Find the length of the diagonals.

b) A plane flies from Sydney on a bearing of 139° for 852 km, then turns and flies on a bearing of 285° until it is due west of Sydney. How far from Sydney is the plane, to the nearest km?
# Mathematics Stage 5.3: Properties of Geometric Figures

**Name:**

**Class:**

## BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

### MEASUREMENT AND GEOMETRY: PROPERTIES OF GEOMETRIC FIGURES

<table>
<thead>
<tr>
<th>STUDENTS HAVE LEARNT TO</th>
<th>SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Write formal proofs of congruent triangles</td>
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<tr>
<td>□ Verify properties of isosceles and equilateral triangles, and special quadrilaterals using congruent triangles</td>
<td></td>
</tr>
<tr>
<td>□ Determine the minimum conditions for triangles to be similar</td>
<td></td>
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<tr>
<td>□ Use deductive reasoning in numerical exercises involving plane shapes</td>
<td></td>
</tr>
<tr>
<td>□ Establish and use the sum of exterior angles and the sum of interior angles results for polygons</td>
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</tbody>
</table>

## STAGE 5.3: PROPERTIES OF GEOMETRIC FIGURES

### SYLLABUS REFERENCE

**MA5.3-16MG** - A student calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar.

### QUESTION 1

Prove that $\triangle MNO$ and $\triangle POM$ are similar.

![Diagram of triangles MNO and POM with side lengths and angles labeled]

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10.

---

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In the diagram below, $OQ = OR = OP$. Prove that $\angle QPR$ is right angled.
STAGE 5.3: PROPERTIES OF GEOMETRIC FIGURES

QUESTION 3

SOLVE PROBLEMS INVOLVING SIMILARITY RATIOS AND AREAS AND VOLUMES

a) The two composite shapes below are similar, find the value of x.

\[ \text{A} = 9 \text{ cm}^2 \quad \text{A} = 16 \text{ cm}^2 \]

b) Two similar objects have volumes 26 cm\(^3\) and 172.75 cm\(^3\) respectively. If the smaller object has a length of 2.5 cm, what will be the length of the larger object to 2 significant figures?
STAGE 5.3: PROPERTIES OF GEOMETRIC FIGURES

QUESTION 4

a) ABCD and AQCP are parallelograms. Prove that BQ = DP.

QUESTION 5

Prove that ∆ABC is equilateral and ∆BCE is isosceles.
Mathematics Stage 5.3: Circle Geometry

Name: 
Class: 

BACKGROUND KNOWLEDGE: STAGE 4 KEY IDEAS

MEASUREMENT AND GEOMETRY: CIRCLE GEOMETRY

STUDENTS HAVE LEARNT TO

☐ Use the language, notation and conventions of geometry
☐ Apply the geometric properties of angles at a point to find unknown angles with appropriate reasoning
☐ Apply the properties of corresponding, alternate and co-interior angles on parallel lines to find unknown angles with appropriate reasoning
☐ Determine and justify that particular lines are parallel
☐ Solve simple numerical exercises based on geometrical properties

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

STAGE 5.3: CIRCLE GEOMETRY

SYLLABUS REFERENCE

MA5.3-17MG - A student applies deductive reasoning to prove circle theorems and to solve related problems.

QUESTION 1 IDENTIFY AND NAME PARTS OF A CIRCLE

Identify the following parts of the circle:

A: 
B: 
C: 
D: 
E: 
F: 
G: 

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STAGE 5.3: CIRCLE GEOMETRY

QUESTION 2

Using the diagram below:

a) identify an angle that subtends from the arc CE. ____________________________________________

b) identify two angles standing on the arc AB. ________________________________________________
STAGE 5.3: CIRCLE GEOMETRY

QUESTION 3

Find the value of the pronumerals in the following and give reasons for your answer:

a) \[
\begin{align*}
\theta &= 56^\circ \\
x &= \text{cm}
\end{align*}
\]

b) \[
\begin{align*}
\alpha &= 43^\circ \\
y &= \text{cm}
\end{align*}
\]

c) \[
\begin{align*}
x &= \text{cm}
\end{align*}
\]

d) \[
\begin{align*}
p &= 5 \text{ cm}
\end{align*}
\]
STAGE 5.3: CIRCLE GEOMETRY

e) \begin{align*}
\angle AOB &= \theta \\
\angle BOC &= 100^\circ \\
\angle COD &= \alpha
\end{align*}

f) \begin{align*}
\angle AOB &= \theta \\
\angle BOC &= 75^\circ \\
\angle BOD &= \beta
\end{align*}

g) \begin{align*}
\angle AOB &= \theta \\
\angle BOC &= 50^\circ \\
\angle COD &= \alpha
\end{align*}

h) \begin{align*}
\angle AOB &= \theta \\
\angle BOC &= 110^\circ \\
\angle COD &= \beta
\end{align*}
QUESTION 4

a) In the diagram below:

i) prove that $\triangle OAC \equiv \triangle OBC$.

ii) hence, prove that $AC = BC$.
b) Find the value of the pronumerals in the following diagrams (provide reasons for your answer):

i)

[Diagram of circle with points A, O, and B, with line segments 6.5 cm, 8 cm, and d.]

\[ x \]  
\[ 6.5 \text{ cm} \]  
\[ 8 \text{ cm} \]  
\[ \beta \]  
\[ \text{d} \]

ii)

[Diagram of circle with points O, x°, and 54°.]

\[ x° \]  
\[ 54° \]  
\[ \beta \]  
\[ O \]
Mathematics Stage 5.3: Single Variable Data Analysis

Name: _____________________________________________

Class: _____________________________________________

BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

STATISTICS AND PROBABILITY: SINGLE VARIABLE DATA ANALYSIS

STUDENTS HAVE LEARNT TO

☐ Determine quartiles and interquartile range
☐ Construct, interpret and use box plots to compare sets of data
☐ Compare shapes of box plots to corresponding histograms and dot plots
☐ Critically evaluate sources of data in media reports and elsewhere

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME

SYLLABUS REFERENCE

MA5.2-18SP - A student uses standard deviation to analyse data.

QUESTION 1

CALCULATING STANDARD DEVIATION

Calculate the standard deviation for the following set of data:

a) (3 5 5 7 9 10 20)

b) [Graph with points at 6, 7, 8, 9, 10]
STAGE 5.3: SINGLE VARIABLE DATA ANALYSIS

QUESTION 2

The following back to back stem-and-leaf plot shows the marks for English and Maths for a class out of 50.

<table>
<thead>
<tr>
<th>MATHS</th>
<th>STEM</th>
<th>ENGLISH</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td>1 4</td>
</tr>
<tr>
<td>8 2</td>
<td>2</td>
<td>2 5</td>
</tr>
<tr>
<td>8 7 5</td>
<td>3</td>
<td>7 7</td>
</tr>
<tr>
<td>9 6 6</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

a) Complete the table for each data set.

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>MEAN</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENGLISH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MATHS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Which subject did the class perform better in? Comment on the spread and mean of each subject.
BACKGROUND KNOWLEDGE: STAGE 5.2 KEY IDEAS

STATISTICS AND PROBABILITY: BIVARIATE DATA ANALYSIS

STUDENTS HAVE LEARNT TO

☐ Construct and interpret displays of bivariate numerical data where the independent variable is time

☐ Construct and interpret scatter plots of two numerical variables

SYLLABUS CONTENT AREAS RELATED TO THE OUTCOME
STAGE 5.3: BIVARIATE DATA ANALYSIS

SYLLABUS REFERENCE

MA5.2-19SP - A student investigates relationships between numerical variables using line of best fit, and explores how data is used to inform decision-making processes.

QUESTION 1

INVESTIGATE BIVARIATE NUMERICAL DATA SETS

The local store kept a record of how much slushies they sell versus the average temperature for the day. The results are displayed below:

<table>
<thead>
<tr>
<th>TEMPERATURE (T)</th>
<th>15°</th>
<th>17°</th>
<th>16°</th>
<th>25°</th>
<th>27°</th>
<th>22°</th>
<th>28°</th>
<th>30°</th>
<th>35°</th>
<th>12°</th>
<th>26°</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLUSHIES SOLD (S)</td>
<td>12</td>
<td>25</td>
<td>47</td>
<td>58</td>
<td>130</td>
<td>60</td>
<td>80</td>
<td>140</td>
<td>70</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

a) enter the data into a spreadsheet and create a scatterplot (use the graph below if you do not have a computer).

b) draw a line of best fit.

c) determine the equation of the line of best fit.

d) determine the value of t, if S = 48.

e) determine the value of S, if t = 29.